PRICE DISPERSION AND LOSS LEADERS

ATTILA AMBRUS AND JONATHAN WEINSTEIN

Abstract. Dispersion in retail prices of identical goods is inconsistent with the standard model of price competition among identical firms, which predicts that all prices will be driven down to cost. One common explanation for such dispersion is the use of a loss-leader strategy, in which a firm prices one good below cost in order to attract a higher customer volume for profitable goods. By assuming each consumer is forced to buy all desired goods at a single firm, we create the possibility of an effective loss-leader strategy. We find that such a strategy cannot occur in equilibrium if individual demands are inelastic, or if demands are diversely distributed. We further show that equilibrium loss-leaders can occur (and can result in positive profits) if there are demand complementarities, but only with delicate relationships among the preferences of all consumers.

1. Introduction

Microeconomics textbooks aside, there is no shortage of evidence of price dispersion, i.e. retail firms charging different prices for identical goods. As Varian (1980) commented, "Economists have belatedly come to recognize that the 'law of one price' is no law at all." One well-known retail strategy which results in price dispersion is the tactic of cutting prices on one good, known as a "loss leader," in order to attract more store traffic and increase profits on other goods. Any price dispersion is, of course, contrary to the unique pure-strategy

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equilibrium prediction in the setting of Bertrand competition that all transactions will take place at marginal cost.[1] This theoretical prediction requires the following assumptions:

1: All firms have identical costs.

2: Consumers have full information, at zero cost, about the prices charged by each firm.

Since the standard argument is given in the context of a market for a single good, when we are in a situation with multiple goods there is another, sometimes hidden, assumption.

3: Consumers are free to buy different goods at different firms, with zero transportation costs, so that the markets for different goods are "uncoupled".

A considerable literature has focused on whether price dispersion can result from relaxing the second assumption, i.e. introducing search costs. In this paper we will keep assumption 2, but will drop assumption 3. In particular, we will go to the other extreme and assume that customers are constrained to purchase their entire bundle of goods from a single firm. This abstracts the idea that it is time-consuming to do one's shopping at multiple locations, essentially changing transportation costs from zero to infinity. Intuitively, one might expect that the resulting "coupled" markets could result in a firm being able to successfully employ a loss-leader strategy, cutting prices on good A below cost in order to attract customers who will buy the profitable good B.

In the single-good case, one proves that there is no equilibrium in which a firm makes positive profits by observing that in this case another firm could "undercut" the profitable firm, charging a slightly lower price and taking all the profits. This argument still has some validity in the multi-good case – now "undercutting" means choosing a price vector which is slightly lower for all goods. The situation is complicated, however, by the fact that when one

[1]There are mixed-strategy equilibria in which prices are above cost and firms make profits. However (in the single-good case) all such equilibria involve unbounded prices and are hence eliminated if consumers have a maximum willingness to pay. In this paper, we will only consider pure-strategy equilibria.

firm undercuts another in an effort to steal its profits, it may attract a different clientele which buys goods in different proportion. If some goods are sold below cost, this may result in losses, confounding the undercutting argument. We will find, however, that under the assumptions in Section 3 it is guaranteed that at least one firm can increase its profits by undercutting another. Our main assumption there is that each consumer has inelastic demand. That is, he demands a fixed vector of goods, which he will buy at whichever firm prices this vector lowest. We also use a genericity assumption on the distribution of demand vectors. This allows us to avoid the technical issue of ties in consumers' firm selection. The distributional assumption also lets us avoid a degenerate case in which two goods are always bought in a fixed proportion, which would allow one price to be raised and the other decreased with no impact on the decisions of any consumers. Under these assumptions, we will find that in equilibrium no one makes any profits, and at least two firms charge exactly marginal cost for all goods, just as in the case of one-good Bertrand competition.

In Section 4 we proceed to the case of more general demands. First we provide an example showing that with a particular specification of demand complementarities, there can be an equilibrium with price dispersion and positive profits. We then show that this example is a rather special case; if consumers are diverse enough, in a particular sense, we prove there can be no profits or price dispersion. The key to the more general impossibility result is a modified form of the undercutting strategy. Now a firm undercuts another firm only on profitable goods, while it slightly increases ("overcuts") the price of subsidized goods. We show that such a strategy steals the profitable consumers of the other firm, while leaving all unprofitable consumers (those who buy too much of the loss-leader goods) with the other firm.

Most of the literature on this topic assumes either bounded rationality, or limited information and search costs. This begs the question of why we want to consider a fully rational model. We feel that it is an important initial step to see whether and to what extent a fully rational, complete-information model can explain the success of a loss-leader strategy. This may further elucidate the role of informational and behavioral considerations. Here, we are able to prove that with sufficient diversity of demands, loss leaders and price dispersion cannot result merely from linking markets through high transportation costs. With specific complementarities, by contrast, loss leaders can be effective in a fully rational world, although the example we use to show this assumes that the set of consumer types is restricted. A possible practical moral to be drawn from this is that (in a full-information, rational world), the loss-leader strategy is apt to work only if it attracts a specific segment of consumers, rather than being an untargeted attempt to increase volume. With limitations on information or rationality, there is more opportunity for loss leaders to succeed, as in the literature we now discuss.

2. Literature Review

Many models have been introduced in which consumers have limited information about prices, whether search costs are introduced explicitly or information is simply limited more directly. In an early paper, Varian (1980) describes a market divided between informed customers, who choose the lowest price, and uninformed customers who choose a store a random. This results in the existence of only mixed-strategy equilibria as firms compromise between attracting the informed customers and exploiting the uninformed.[2] Therefore the model predicts that price dispersion will persist, but each firm's prices will fluctuate and no firm will consistently price lower or higher than others. This contrasts with the pure-strategy price dispersion we find in Section 4.1.

Lal and Matutes (1994) analyze a model with advertising, in which consumers must decide where to shop knowing only the prices of goods which the stores choose to advertise, and only observe the remaining prices once at the store. The consumers have rational expectations and therefore anticipate that unadvertised goods will be overpriced; nevertheless, in equilibrium both stores

[2]In a more recent paper, Kamenica (2008) analyses the menu choice decision of a monopolist when a fraction of consumers are uninformed about the quality of the goods, and shows that the presence of these consumers may induce the firm to introduce premium loss leaders that are unprofitable on their own but increase the demand for other goods.

do employ a loss-leader strategy, advertising a particular good which is priced below cost. More recently, Spiegler (2006) analyzes a model with limited information and a simplified decision rule for consumers. In this case, consumers randomly observe one price from each store and choose the store for which that one price provides the highest consumer surplus. This boundedly rational choice procedure, called $S(1)$, was introduced by Osborne and Rubinstein (1998). Spiegler finds that firms make positive profits in equilibrium. Also, the variance in prices increases as the number of firms increases, contrary to the usual intuition about competition. This is essentially because with a very large number of firms, the best way to get attention from $S(1)$ consumers is to have a small number of goods with an extremely low price. In fact, if the model did not include a hard lower bound on prices, the price of the loss leaders would become arbitrarily negative as the number of firms grew. An alternative model of limited search is analyzed by Chen, Iyer and Pazgal (2007) who assume that consumers have limited memories for prices. In particular, the consumers divide the set of possible prices into finitely many ranges and only remember which range a price is in. They find that this limited memory enables firms to extract surplus. Finally, Eliaz and Spiegler (2007) study a model in which firms use costly marketing devices to influence the set of alternatives that boundedly rational consumers consider to be relevant. They identify mixed strategy equilibria in which firms with positive probability offer goods that are not chosen by consumers but help attract consumers from rival stores. These goods are loss leaders since including them on the menu is costly.

The role of this paper is to examine the possibility of price dispersion in a world with full information. Since we find it can occur only in certain special circumstances, it is reasonable to think that loss leaders are more often supported by limits on information.

3. Inelastic Demand

3.1. **Model.** We have $K > 1$ identical firms, each of which sells N goods, each with constant marginal cost which we normalize to zero (so that prices should be interpreted throughout as the difference from marginal cost.) There is a continuum of consumers of mass 1, with each consumer having inelastic demand, so that each individual's demand is characterized by a non-negative vector in $\mathbb{R}^N$, specifying the quantity he purchases of each good. The distribution of demands is described by a probability distribution P on $\mathbb{R}^N$ with support in the non-negative orthant. We assume that P assigns zero mass to any hyperplane in $\mathbb{R}^N$. The consumers, who are only able to shop at one firm, select a firm which minimizes their cost. A profile $(\mathbf{p}_1, ..., \mathbf{p}_K)$ of price vectors is an equilibrium if no firm could increase profits by changing prices (consumers then adjusting their firm choice.) As mentioned in the introduction, we will only consider pure-strategy equilibria.

We are interested in whether there exists a pure-strategy equilibrium in which firms earn positive profits. Note that if $N = 1$ we are in the case of standard Bertrand competition, and the only equilibrium outcome is for all firms to make zero profits, with at least two firms charging exactly marginal cost.

A few words are in order on the assumption that hyperplanes have zero mass. Note that a distributional assumption is clearly necessary to exclude price dispersion. In particular, if two goods were always demanded in a fixed ratio, firms could always increase one price and decrease the other without any effect on equilibrium. Also, it is clear any distribution can be slightly perturbed to give one satisfying the assumption.

3.2. **At-Cost Pricing Result for Inelastic Demands.**

Note that it is clearly an equilibrium for all firms to set all prices equal to zero, or even for at least two firms to do so and others to charge non-negative prices. In such a case all firms make zero profits and no one can do better. In this section we will prove that under the assumptions given above, these are the only equilibria.

Theorem 1. *In any equilibrium, (a) all firms make zero profits, (b) at least two firms have all prices equal to marginal cost, and (c) all customers go to such firms.*

Proof of (a). First note that no firm can make negative profits in equilibrium, because it can always assure itself of non-negative profits by charging at or above cost for all goods. Now suppose we have an equilibrium $(\mathbf{p}_1, ..., \mathbf{p}_K)$. Let $S_i = \left\{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} < \mathbf{p}_j \cdot \mathbf{x}, \forall j \neq i\right\}$ be the set of consumer demands for which firm i is preferred to all other firms, and also let $S_{i,j} = \left\{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} < \mathbf{p}_j \cdot \mathbf{x} < \mathbf{p}_k \cdot \mathbf{x}, \forall k \neq i, j\right\}$be the set of demands for which i is preferred and j is second-best. Each firm's profits are given by $\pi_i = \int_{S_i} \mathbf{p}_i \cdot \mathbf{x} \; dP(\mathbf{x})$. Define $\mathbf{1}$ as the vector $(1, ..., 1) \in \mathbb{R}^N$. Let $UC_{i,j,\epsilon} = \left\{\mathbf{x} \in \mathbb{R}^N : (\mathbf{p}_j - \epsilon \cdot \mathbf{1}) \cdot \mathbf{x} < \mathbf{p}_k \cdot \mathbf{x}, \forall k \neq i, j\right\}$ be the subset of demand space for which firm i is chosen after it undercuts firm j by switching to price vector $\mathbf{p}_j - \epsilon \cdot \mathbf{1}$. Also define $\pi_{i,j,\epsilon}$ as the profit of firm i if it made this undercutting deviation. Assume at least one $\pi_i > 0$. As discussed in the introduction, it is not automatically profitable to undercut a profitable firm. We will be able to show, however, that we are not at equilibrium by showing that for *some* triple (i, j, ϵ), $\pi_{i,j,\epsilon} > \pi_i$.

We will first consider the case in which some two firms i and j offer identical price vectors. If firms i and j make positive profit, either one could double its profits by decreasing prices slightly and attracting all consumers who go to these two firms (the argument also applies if there is a tie among more than two firms.) If firms i and j make zero profits and some other firm k makes positive profits, it will certainly be profitable for firm i to undercut firm k – since the price vector $\mathbf{p}_i$ is still available from firm j, firm i will only attract consumers from the set S_k and no others, so the deviation will be profitable.

Henceforth we assume all price vectors are distinct. Notice that for any i and j, the set of firms for which i and j are preferred equally is the hyperplane $\left\{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} = \mathbf{p}_j \cdot \mathbf{x}\right\}$ which is assumed to have mass zero, so we can safely ignore the issue of ties. If firm i undercuts firm j, it will be selected by all consumers who previously chose firm j. It will also be chosen by those who previously chose firm i but liked firm j second best, because firm i's old price vector is no longer available to them. That is, $S_j \cup S_{i,j} \subset UC_{i,j,\epsilon}$. Any $\mathbf{x} \notin S_j \cup S_{i,j}$ satisfies $\mathbf{p}_k \cdot \mathbf{x} \leq \mathbf{p}_j \cdot \mathbf{x}$ for some $k \neq i, j$. When the inequality is strict,

$\mathbf{p}_k \cdot \mathbf{x} < (\mathbf{p}_j - \epsilon \cdot \mathbf{1}) \cdot \mathbf{x}$ for small enough ϵ, proving that $UC_{i,j,\epsilon} - (S_j \cup S_{i,j})$ converges as $\epsilon \to 0$ to a set where some preferences are tied. By Lemma 1 this implies that $P(UC_{i,j,\epsilon} - (S_j \cup S_{i,j})) \to 0$ as $\epsilon \to 0$. This in turn implies that $\pi_{i,j,\epsilon} \to \int_{Sj \cup S_{i,j}} \mathbf{p}_j \cdot \mathbf{x}\, dP(\mathbf{x})$ as $\epsilon \to 0$.

Assume without loss of generality no firm makes smaller profit than firm 1 in our equilibrium. It will suffice to show that for some j, $\int_{S_j \cup S_{1,j}} \mathbf{p}_j \cdot \mathbf{x}\, dP(\mathbf{x}) > \pi_1$, for then there will exist an ϵ for which $\pi_{1,j,\epsilon} > \pi_1$. Observe that

$$\begin{aligned}
\sum_{j=2}^{N} \int_{S_j \cup S_{1,j}} \mathbf{p}_j \cdot x dP(\mathbf{x}) &= \sum_{j=2}^{N} \left[\int_{S_j} \mathbf{p}_j \cdot x dP(\mathbf{x}) + \int_{S_{1,j}} \mathbf{p}_j \cdot x dP(\mathbf{x}) \right] \qquad (1) \\
&\geqslant \sum_{j=2}^{N} \left[\pi_j + \int_{S_{1,j}} \mathbf{p}_1 \cdot x dP(\mathbf{x}) \right] \\
&= \sum_{j=2}^{N} \pi_j + \int_{\cup_j S_{1,j}} \mathbf{p}_1 \cdot x dP(\mathbf{x}) \\
&= \sum_{j=2}^{N} \pi_j + \int_{S_1} \mathbf{p}_1 \cdot x dP(\mathbf{x}) \\
&= \sum_{j=1}^{N} \pi_j
\end{aligned}$$

where the inequality comes from the revealed preference of customers in the set $S_{1,j}$; we know they pay less at firm 1 than at firm j.

Because there are $N-1$ terms in the left-hand sum, the above inequality implies that for at least one j, $\int_{Sj \cup S_{i,j}} \mathbf{p}_j \cdot \mathbf{x}\, dP(\mathbf{x}) \geq \frac{\sum_{j=1}^{N} \pi_j}{N-1} > \frac{\sum_{j=1}^{N} \pi_j}{N} \geq \pi_1$, where the strict inequality is implied by at least one $\pi_j > 0$.[3] Then firm 1 can improve its profit by undercutting firm j. This proves (a).□

The inequality in (1) is precisely the stage of the proof at which we made use of the individually inelastic demands. In particular, this assumption implied that since firm 1's customers, in aggregate, pay a non-negative surplus to firm 1, they would continue to pay a non-negative surplus if firm 1 vanished and

[3] The very last inequality is not strict in the case that all π_i are equal. This is not a concern since the previous inequality is strict.

they had to patronize their second choice. This is effectively what happens as we consider the outcome of firm 1 undercutting each of the other firms in turn; when he undercuts firm j he attracts not only firm j's old customers, but also his old customers who liked j second best. Our assumption on demands ensure that on *average*, across all firms he might undercut, this does not hurt him.

Part (a) of the theorem leaves open the possibility of an equilibrium in which firms make profits on some customers but losses on others, but we will now see that this is not possible.

Claim 1. *In any equilibrium, the set of consumers who pay exactly the marginal cost of their bundle has mass 1.*

Proof. We will show that if this is not the case, then for some ϵ, any firm could make positive profits by switching to price vector $\epsilon \cdot \mathbf{1}$. Let $S = \left\{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} > \mathbf{0}, \forall i\right\}$ be the set of demands for which the best price available is above the cost of the bundle. Let $S_\epsilon = \left\{\mathbf{x} \in \mathbb{R}^N : (\epsilon \cdot \mathbf{1}) \cdot \mathbf{x} < \mathbf{p}_i \cdot \mathbf{x}, \forall i\right\}$ be the set of demands for which the prices $\epsilon \cdot \mathbf{1}$ are preferred to those currently available. Note that $S = \cup_{n=1}^{\infty} S_{\frac{1}{n}}$.

Note that if $P(S_\epsilon) > 0$, then any firm can make profits by switching to price vector $\epsilon \cdot \mathbf{1}$, because it would attract consumers in S_ϵ and all customers are paying above cost. Therefore, $P(S_\epsilon) = 0$. Then by countable additivity, $P(S) = P(\cup_{n=1}^{\infty} S_{\frac{1}{n}}) \leqslant \Sigma_{n=1}^{\infty} P(S_{\frac{1}{n}}) = 0$. Therefore, the set of customers who pay at most the cost of their bundle has mass 1. Suppose a non-zero mass of customers are paying below cost. Then, some firms would have to be suffering losses, but this cannot happen in equilibrium. □

This result still leaves open the possibility that although all consumers pay the cost of their bundle, some goods are priced above cost and others below. We will now exclude this possibility – the assumption that hyperplanes have zero mass is needed again here.

Proof of (b) and (c). Note that if $\mathbf{p}_i \neq \mathbf{0}$, then the set $\left\{\mathbf{x} \in \mathbb{R}^N : \mathbf{p}_i \cdot \mathbf{x} = \mathbf{0}\right\}$ is a hyperplane and so has mass zero by assumption. Together with Claim 1 this immediately gives (c). Therefore, we must have $\mathbf{p}_i = \mathbf{0}$ for some i. If this were true only for firm i, we could repeat the argument in the proof of Claim 1 to show that firm i could deviate and make profits by choosing an appropriate vector $\epsilon \cdot \mathbf{1}$, so it must hold for at least two firms, proving (b). $\square$

4. Elastic demand

4.1. **An example with positive profits.**

Below we provide an example which shows that if demands are not perfectly inelastic, the game can have equilibria in which stores earn strictly positive profits. The example involves a relatively complex structure of complementarity relationships among different goods, and particular sets of consumer types. This raises the question of how relevant examples like these are in practice. In the next subsection we provide an assumption on diversity of consumer demands that we show leads to zero profits, and therefore rules out examples like the one discussed here. The upshot is that positive profits are possible, but only with specific delicate relationships among the consumer demands.

Before we state the example formally, here is a summary of the idea. There are two stores and six goods, and in the proposed equilibrium one store ends up selling only the first three, the other store only the second three. There are three types of consumers going to the first store. One type only buys the first good and has a high reservation value for it. For the second type of consumers, the first two goods are perfect complements, and they buy a unit of each. Finally, for the third type of consumers, the first three goods are perfect complements, and they buy a unit of each. Moreover, the reservation value of the second type of consumers for a pair of goods 1 and 2 is lower than the reservation value of the first type of consumers for only good 1, and the reservation value of the third type of consumers for a basket of all three goods is even lower than the former. This implies that the optimal way for the store to sell these goods is to set a high price for the first good, and subsidize (sell below marginal cost) the second and third goods: this way a large profit can be extracted from the first type of consumers, and at the same time positive profit can be extracted from the other two types of consumers. The other store prices its goods symmetrically, and ends up selling the other three types of goods, to the other three consumer types. The key feature of the example is that given this pricing structure, in which two of the goods are sold at a subsidized price, the stores do not want to attract the consumers of the other store. This is because these types are constructed such that they do not value the good that generates the profit to the store, but they would purchase relatively large quantities of the subsidized "loss-leader" goods. That is, stores in the equilibrium we propose could tempt over consumers from the other store, by undercutting the equilibrium prices of the goods currently sold by the rival, but they do not find it profitable, because these consumers generate more loss than gain given the optimal pricing structure for the goods already sold in the store.[4]

Example 1. As above, let the marginal cost of all goods be normalized to 0. There are 6 goods: $a_1, a_2, a_3, b_1, b_2, b_3$, 6 consumer types: A_1, A_2, A_3, B_1, B_2, B_3, and two stores: I, II. There is a measure 2 of each of type A_1 and type B_1 consumers, a measure 3 of each of type A_2 and type B_2 consumers, and a measure 14 of each of type A_3 and type B_3 consumers.

For type A_1 consumers, the reservation value for the first unit of good a_1 is 170, the reservation value for the first twenty units of b_2 is -0.5 (meaning that these consumers buy twenty units of b_2 if the price falls below marginal cost minus 0.5 monetary units), the reservation value for the first twenty units of b_3 is -0.5, and the reservation value of any other units of any good is $-K$ (where K is a "large" number).

For type A_2 consumers, the first unit of a_1 and of a_2 are perfect complements. In particular, the reservation value for the first units of a_1 and a_2 together is 164 (the reservation value for only one of these goods without the other is $-K$). The reservation value for the first twenty units of b_2 and b_3 are -0.5 each, and the reservation value for any other units of any good is $-K$.

[4]This construction is similar to the one presented in Ambrus and Argenziano (08), showing that in two-sided markets with network externalities Bertrand competition among platforms is consistent with positive profits in equilibrium, provided that platforms subsidize consumers on one side of the market, and generate positive profit on the other side.

For type A_3 consumers, the first units of a_1, a_2, a_3 are perfect complements. In particular, the reservation value for the first units of a_1, a_2 and a_3 together is 161 (the reservation value for any unit of single good or any pairs of goods without the third one is $-K$). The reservation value for the first twenty units of b_2 and b_3 are -0.5 each, and the reservation value for any other units of any good is $-K$.

Types B_1, B_2, B_3 have preferences symmetric to the above, with the roles of a_i and b_i interchanged.

Claim 2. *The following profile constitutes an equilibrium. Firm I sets prices $p(a_1) = 10$, $p(a_2) = -6$, $p(a_3) = -3$ and $p(b_1) = p(b_2) = p(b_3) = 300$, while II sets prices $q(a_1) = q(a_2) = q(a_3) = 300$, $q(b_1) = 10$, $q(b_2) = -6$ and $q(b_3) = -3$. Consumers of type A_1 go to I and each buys one unit of of a_1, consumers of type A_2 go to I and each buys one unit of of a_1 and one unit of a_2, type A_3 consumers go to I and buy one unit of each a_1, a_2 and a_3. Consumer types B_1, B_2 and B_3 go to II and behave symmetrically (buy one b_1, one b_1 and one b_2, and one b_1, b_2 and b_3 respectively).*

Proof. First, note that each type of consumer gets a consumer surplus of 160 in the profile specified above, which is exactly how much they could get if switching to the other store. Hence, consumers are in optimum.

Next we check if the stores have any profitable deviation. Since the profile and the game is symmetric, without loss of generality we check for deviations by store I. Note that in the above profile the profit of store I is 46 (20 from type A_1 consumers, 12 from type A_2 consumers, and 14 from type A_1 consumers).

First, consider deviations in which firm I does not set any of the prices $p(a_2)$, $p(a_3)$, $p(b_2)$, $p(b_3)$ below -0.5. Then consumers of types A_i do not purchase goods b_j, and consumers of types B_i do not purchase goods a_j, and the optimal prices for a_j and b_j can be chosen separately. It is easy to see that among prices that only attract consumers of type A_1 (and not A_2 or A_3) to the store, the ones generating the highest profit involve $p(a_1) = 10$. This generates a profit of 20. Among prices that only attract consumer types A_1 and A_2, the ones generating the highest profit involve $p(a_1) = 4.5$ and $p(a_2) = -0.5$. This generates a profit of 21. Finally, among prices that attract all types A_i, the one generating the highest profit is $p(a_1) = 2$, $p(a_2) = -0.5$ and $p(a_3) = -0.5$. This generates a profit of 22.5. It is easy to see that attracting different subsets of consumers among types A_i are either suboptimal or infeasible with nonnegative prices. Symmetric considerations hold for prices $p(b_1), p(b_2), p(b_3)$ and types B_i. This implies that the highest amount of profit store I can achieve through prices that are not below -0.5 is 45.

Next, observe that any consumer going to store II could get a surplus of 160, store I is extracting the highest possible profit from consumer types A_i. Similarly, given the prices of store II, store I cannot extract a profit of more than 46 from consumer types B_i. Hence, for a profitable deviation it is necessary that the store attracts consumers both from types A_i and from types B_i.

The above observations establish that either (i) $\min(p(a_2), p(a_3)) < -0.5$ and store I attracts some consumers among types B_i; or (ii) $\min(p(b_2), p(b_3)) < -0.5$ and store I attracts some consumers among types A_i. The cases are symmetric, so assume case (i). Then any consumer of type B_i who goes to store I buys 20 units of either a_2 or a_3. This generates a loss of at least 10 for the store, implying that none of these consumers can generate positive profit for the store. Therefore, the deviation cannot be profitable, since the profit that consumer types A_i generate for the store cannot be higher than 46. This in turn implies that there is no profitable deviation for the store. □

4.2. **Zero-profit result for broad-demand case.** The previous subsection demonstrated that if demand is elastic and there are complementarities among goods then there can be equilibria in which stores obtain strictly positive profit. Below we show that this cannot be the case if the set of consumer types is diverse enough: the unique equilibrium in this case entails that all goods are sold exactly at marginal price, and all stores obtain zero profit. The pricing strategy that we use to show this result is not a simple undercutting strategy (that is, undercutting the rival's price for all goods), but an undercutting-overcutting strategy, which undercuts the rival at goods whose prices are above marginal cost and at the same time raises the price of subsidized goods by an appropriate proportion.

In the general case consumers cannot be simply described by consumption bundles they want to purchase, as in the previous section. Instead, consumers are described by their preferences over consumption bundles. For this reason, let the consumption space be $\mathbb{R}_+^{N+1}$, where the first N dimensions are associated with the N goods sold by the stores, and the last dimension is associated with a numeraire good (money). We assume that all consumers have quasi-linear preferences in money. For technical convenience, we also assume that the Walrasian demand of every consumer is single-valued and continuous in prices.[5] That is, fixing the choice of store of a consumer, her demand is a continuous function of prices of that store. A sufficient condition for this is that the preferences of consumers are continuous and strictly quasi-concave. The example in the previous subsection does not satisfy this property, but this distinction is not the essential one. The example could be modified such that demand functions of all consumers are continuous and the equilibrium with positive profits prevails.

To summarize, our consumers have preference relations on $\mathbb{R}_+^{N+1}$ that are quasi-linear in the numeraire good and exhibit a continuous demand function in prices. Let P denote the distribution of consumer types over this set. As before, we normalize the marginal costs of all goods to be zero. Note that in any game in this context, a profile in which all stores set all their prices to zero constitutes an equilibrium. There can be no profitable deviation, because no consumer would choose to pay above zero. Hence, existence of pure strategy equilibrium is not an issue.

Our next result, which holds without any further assumption on the distribution of consumer types, establishes that in any equilibrium, all K stores have the same profit. The basic idea is that given an arbitrary strategy profile, any

[5]This is a stronger requirement than what is needed for our main result. It would be sufficient to assume that the aggregate demand of any set of consumers with nonzero measure is a continuous function. This is shown to be a generic property of economies: see Ichiishi (76).

store can choose a deviation strategy such that its resulting profit level arbitrarily approximates any other store's profit in the previous profile. As the example in the previous section suggests, this is not always possible using a strategy that undercuts the prices of another store for all goods: some goods might be subsidized by the other store, and undercutting the rival might attract a set of consumers that contain too many of those who mainly buy the subsidized goods and thereby generate losses to the store. In particular, while store A slightly undercutting store B's prices does imply stealing all the former consumers of store B, among the original consumers of store A there might be more of those consumers sticking with store A who generate a loss under the new prices. However, below we show that there exists a slightly different deviation, which only steals the profitable consumers of store B, and at the same time makes sure that only those old consumers of store A stick with the store who are profitable under the new prices. The key idea is to slightly lower the prices for goods that store B sells above marginal cost, while slightly increasing the prices for goods that store B sells below marginal cost, so that the relative magnitudes of these price changes are proportional to the absolute values of the prices. Note that for this infinitesimal change, those customers of store B who prefer the new set of prices to the old prices of store B are exactly those who purchase bundles that are on average profitable to the store. Similarly, among existing consumers of store A, only those who would buy profitable bundles might stick with A, while the rest of these consumers either switch to B or some other store.

Proposition 1. *For any distribution of consumers P, the profits of all stores are equal in any equilibrium.*

Proof. Suppose that in some equilibrium firm j's profit is strictly higher than firm i's profit. Note that this in particular means that firm j's profit is strictly positive, which in turn implies that at least one of its prices is strictly positive. Consider the deviation $(1-\epsilon)\cdot\mathbf{p}_j$ by firm i, for $\epsilon > 0$. Then, any consumer who in the candidate equilibrium profile chose firm j and purchased a bundle for strictly positive price now chooses i. To see this, note that the price of any bundle $\mathbf{x}$ at store i after the deviation is $(1-\epsilon)\cdot\mathbf{p}_j\mathbf{x}$, which is strictly less than $\mathbf{p}_j\mathbf{x}$ if the latter is strictly positive. Hence, the previously optimal bundle for former consumers of j who spent positive amount of money in the store is attainable at store i for strictly less money. This means that these consumers strictly prefer store i after the deviation to any of the other stores. By assumption, the aggregate demand of these consumers is continuous in prices, hence as ϵ goes to zero, the revenue of store i from these consumers goes to the revenue of store j from the same consumers in the candidate equilibrium profile. By definition of these consumers, the latter limit revenue is weakly larger than store j's profit in the candidate equilibrium profile. Finally, observe that any other consumer who after the deviation chooses store i spends a nonnegative amount of money. This is because if $(1-\epsilon)\cdot\mathbf{p}_j\mathbf{x}<0$ then $\mathbf{p}_j\mathbf{x}<(1-\epsilon)\cdot\mathbf{p}_j\mathbf{x}$, hence bundles that can be purchased for negative amount of money can be obtained even cheaper at store j. This concludes that for any $\delta>0$, for small enough $\epsilon>0$ the above deviation yields a profit to i that is at least firm j's profit in the candidate equilibrium profile minus δ. This contradicts that the candidate profile is an equilibrium, since in the candidate profile firm i's profit is strictly smaller than form j's profit. □

We now turn our attention to the possible existence of an equilibrium with equal and positive profits, as in the previous section. We will show that if the set of consumer types is diverse then such equilibria are not possible. We define diversity of the set of consumers the following way: for any set of prices and any combination of goods, there exists a mass of consumers who demand a positive amount of each of those goods, but not of any other good. In particular, and these are the key implications of the assumption that we use in proving the next proposition, the following hold: (i) for any good n, there is a set of consumers with positive mass who only want to purchase good n, no matter which store they go to; and (ii) there is a set of consumers with positive mass that want to purchase a positive amount of every good, no matter which store they go to.

A1: For any K-tuple of prices $\langle\mathbf{p}_1,...,\mathbf{p}_K\rangle$, and any set of goods $N'\subseteq\{1,...,N\}$, there is a positive mass of consumers who at each of the prices $\mathbf{p}_1,...,\mathbf{p}_K$ demand a strictly positive amount exactly from goods in N'.

Theorem 2. *If A1 holds then in any equilibrium (a) all firms make zero profits, (b) at least two firms have all prices equal to marginal cost, and (c) all firms set all prices weakly above marginal cost.*

Proof. Consider first the case that some firm chooses a negative price in equilibrium. Let n be a good such that at least one store charges a negative price for it. By A1 there is a set of consumers with positive mass, who at all of the prices chosen by the stores in the proposed equilibrium would only purchase n. These consumers obviously choose one of the stores charging a negative price for n. Then there exists a store i such that it charges a negative price for n, and it attracts a positive mass of consumers who only buy good n. Then the amount of profit generated by the profitable consumers of store i is strictly larger than the equilibrium profit level of i. Denote this set of consumers by S_i^*. Consider now the deviation $(1-\epsilon)\mathbf{p}_i$ for $\epsilon > 0$, by any firm $j \neq i$. As argued in the proof of Proposition 2, this deviation steals all consumers in S_i^*, and it does not attract any consumer who would spend a negative amount in the store. By assumption the aggregate demand by S_i^* is continuous in prices, hence as ϵ converges to zero, the profit generated by S_i^* converges to the level that these consumers generate at prices $\mathbf{p}_i$. Therefore, for small enough ϵ the above deviation is profitable. This proves (c).

Consider now the case that all firms only charge nonnegative prices, but profit levels are strictly positive. This in particular implies that there is a good for every store such that the price charges a strictly positive price for the good and the demand for the good by consumers of the store is strictly positive. By A1, there is a set of consumers S^0 with positive mass, who at each of the price vectors $\mathbf{p}_1, ..., \mathbf{p}_K$ set by the stores in the proposed equilibrium would purchase a strictly positive amount of each of the goods. For any two firms i and j, let $S_{i,j}^0$ denote the subset of S^0 containing consumers for whom the first choice at prices $\mathbf{p}_1, ..., \mathbf{p}_K$ is firm i, and the second choice is firm j. Since there is a finite number of firms, $S_{i,j}^0$ has a positive mass for some firms i, j. Note that by definition consumers in $S_{i,j}^0$ demand a positive amount of each good at prices $\mathbf{p}_j$, including the ones whose prices are set to be strictly positive by j. Hence, at these prices consumers in $S^0_{i,j}$ generate a profit $\pi^0_{i,j} > 0$. Consider now a deviation $\mathbf{p}_j - \epsilon \cdot \mathbf{1}$ by firm i, for $\epsilon > 0$. This deviation in particular attracts all former consumers of firm j, and consumers in $S^0_{i,j}$. Since the aggregate demand of this combined set of consumers is continuous in prices, as ϵ goes to zero, the profit obtained from this set of consumers after the deviation by firm i converges to the equilibrium profit level of firm j plus $\pi^0_{i,j}$. Moreover, the profit obtained from other consumers converges to a nonnegative amount, since $\mathbf{p}_j \geq \mathbf{0}$. Hence, for small enough ϵ the proposed deviation is profitable, contradicting that the profile constitutes an equilibrium. This proves (a).

Suppose now that there is at most one firm that sets all prices to be zero. Let i be a store such that there is no other store who charges prices $\mathbf{0}$. By A1, there is a positive mass of consumers who at each of the prices in the above profile would purchase a positive amount of all goods. Then there is a store $j \neq i$, and a subset of these consumers $S_{-i,j}$ with positive mass whose choice among stores excluding i would be j. Store i then has a profitable deviation by setting its prices to be $\frac{1}{2}\mathbf{p}_j$. This deviation attracts all consumers in $S_{-i,j}$, each of whom purchase a positive amount of all goods. By assumption $\mathbf{p}_j \geq 0$ and $\mathbf{p}_j \neq 0$, hence the above implies that the deviation yields strictly positive profit to store i, contradicting that in equilibrium all stores have zero profit. This proves (b). $\square$

Note that the example in the previous subsection violates assumption A1. In particular, at the prices set by the two firms in the proposed equilibrium, no consumers would like to buy only the subsidized goods a_2, a_3, b_2 or b_3. If there was a positive mass of such consumers, no matter how small this mass was, the firms would have a profitable deviation. This is because the above consumers would be unprofitable for the stores, and hence the same undercutting-overcutting strategy that we used to show equality of profits would strictly improve a firm's profit, by stealing all the profitable consumers of the other store, and getting rid of the unprofitable ones.

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Harvard University and Northwestern University