Supplementary Appendix: For Online Publication Only

Appendix A. Generalized Theoretical Results

In this section we examine under what conditions our main conclusions extend to more general utility functions, endowment distributions and surplus division rules. The environment with CARA utilities and jointly normally distributed endowments facilitates a convenient transferrable (expected) utilities environment that is particularly tractable to analyze when social surplus is divided in accordance with the Myerson value. While analytical tractability requires a series of strong assumptions, below we show that some of the main qualitative insights of the model extend to much more general specifications.

For general specifications of the model utilities are nontransferable and social surplus (as a single number) is undefined, hence we need a more general approach. Let v_i be the utility function for agent i, mapping second period consumption into utility. We assume that $v_i = v_j$ for all i and j in the same group, and that v_i is strictly increasing and strictly concave for all $i \in \mathbf{N}$. These properties imply that for any number of agents more than one, and for any point of the Pareto frontier of feasible consumption plans that can be reached via risk-sharing arrangements, there is a direction along the Pareto frontier in which a given agent's expected utility is strictly increasing. Let $\mathcal{P}_k$ be the endowment distribution of agents in group $k \in \mathbf{M}$.

Let $\mathcal{L}$ be the set of all possible networks for agents in $\mathbf{N}$. We assume that agents correctly foresee what risk-sharing arrangement they would agree upon for any possible $L \in \mathcal{L}$. These risk-sharing arrangements, which depend on the social network, might be dictated by social conventions, or they can be outcomes of negotiation processes for transfer arrangements once the network is formed. Let $\tau(L)$ be the transfer arrangement and $u_i^\tau(L)$ be the expected second period consumption utility of agent i implied by $\tau(L)$.[47] We refer to $\tau(\cdot)$ as the surplus division rule.

We assume that for every $L \in \mathcal{L}$, $\tau(L)$ specifies a pairwise-efficient risk-sharing arrangement $\tau_{ij}(L)$ for every pair of agents i, j linked in L. As shown earlier, this is equivalent to $\tau(L)$ being Pareto efficient at a component level.

Agent i maximizes the difference between expected utility from the second period risk sharing (given by u_i^τ) and her costs of establishing links.

Let $C_i(L)$ be the set of agents on the same component as i given L, and recall that G is a function mapping agents in $\mathbf{N}$ to groups in $\mathbf{M}$.

Next we impose a series of assumptions on $\tau(\cdot)$. We do not claim that the above assumptions hold universally when informal risk-sharing takes place, but they are relatively weak requirements that are natural in many settings. Our main objective is to demonstrate that our qualitative results hold for a much broader class of models than the CARA-normal setting with surplus division governed by the Myerson value.

[47] More precisely, utility function v_i, the distribution of endowment realizations and transfer arrangement $\tau(L)$ jointly determine $u_i^\tau(L)$.

The first assumption requires that establishing a link always strictly increases the connecting agents' expected consumption utilities.

Assumption 12. *$u_i^\tau(L \cup \{l_{ij}\}) > u_i^\tau(L)$ for every $L \in \mathcal{L}$, $i, j \in \mathbf{N}$ and $l_{ij} \notin L$.*

The next assumption requires that establishing an essential link does not impose a negative externality on other agents. This implies that while both i and j privately benefit from essential link l_{ij}, in terms of second period expected utility, they do not benefit over and beyond the enhancement of risk-sharing opportunities that the link facilitates.

Assumption 13. *$u_k^\tau(L \cup \{l_{ij}\}) \geq u_k^\tau(L)$ for every $L \in \mathcal{L}$, $i, j, k \in \mathbf{N}$ and $C_i(L) \neq C_j(L)$.*

Next we extend the idea that the private benefit that two agents receive from establishing a link should be increasing in the distance between them in the absence of the link. In the previous analysis these private benefits depended specifically on the Myerson distance between the two agents, while here we allow for a general class of distance measures. Before defining the class of distance measures we allow for, some additional notation is required. For two sets S and S' we define $\mathcal{M}(S, S')$ as the set of matching functions $\mu : S \to S' \cup \{\emptyset\}$, such that for $s \in S$ if $\mu(s) \neq \emptyset$ then $\mu(s) \neq \mu(t)$ for all $t \in S \setminus \{s\}$. Thus every $\mu \in \mathcal{M}(S, S')$ maps each element of S into a different element of S', or else the empty set.

Let $\overline{\mathbf{N}}^2 = \{(i, j) | i, j \in \mathbf{N}, i \neq j\}$.

Definition (Distance measure): A distance measure is a mapping $d : \overline{\mathbf{N}}^2 \times \mathcal{L} \to \mathbb{R}_{++}$ satisfying the following properties:

Assumption 14.

(i) *If i and j are in different components on L, then $d(i, j, L) = \overline{d}$, with $\overline{d}$ strictly greater than the maximum possible distance between any two path-connected agents.*

(ii) *The distance measure depends only on paths (thus ignoring walks with cycles).*

(iii) *Let S_{ij} be the set of paths between i and j and S_{kl} be the set of paths between k and l. We assume $d(i, j; L) > d(k, l; L)$ if there exists a matching function $\mu \in \mathcal{M}(S, S')$ such that each path between i and j is matched to a shorter path between k and l, and all such paths between k and l are independent (do not pass through any of the same nodes as each other).*

Assumption 14 places only weak restrictions on the distance measure. In particular, part (iii) in general only provides a very weak partial ordering of the distances between agents. However, there is a special case in which the ordering is complete. On a tree network, there is a unique path between any two agents, so this determines the ordering of distances between pairs of agents. In what follows, let $d(\cdot)$ be any distance measure satisfying the above requirements.

While we will use the concept of distance between agents in the general case of multiple groups, first we focus on extending our earlier results for the case of homogeneous agents. Next we make assumptions on how distance in the absence of a link influences the private benefits of two agents within the same group establishing that link.

The next assumption requires that if all agents are from the same group then the private benefit two agents receive when establishing a link only depends (positively) on their distance in the absence of the link, and on the sizes of the components they are on. Recall that in our benchmark model in the CARA-normal setting these private benefits only depended on the Myerson-distance between the agents. The requirement below allows the private benefit to depend on different distance measures, and also on the sizes of the agents' components (which for general utilities influences the difference between the Pareto frontiers of feasible consumption plans with and without the link).

Assumption 15 (Only Distance and Size Matter)**.** *If $G(i) = G(j)$ for all $i, j \in \mathbf{N}$ and $l_{ij} \notin L$, then*

$$u_i^\tau(L \cup \{l_{ij}\}) - u_i^\tau(L) = g(d(i,j,L), |C_i(L)|, |C_i(L \cup \{l_{ij}\})|),$$

Moreover, $g(d(i,j,L), |C_i(L)|, |C_i(L \cup \{l_{ij}\})|)$ is increasing in $d(i,j,L)$.

As Assumption 15 does not apply when there are multiple groups, in the multiple group case the composition of each component, in terms of the groups the constituent agents come from, and their network positions, can matter.

The last assumption we need for recreating the results of the benchmark model for homogeneous agents is that the cost of link formation within a group is sufficiently small relative to the private benefits from establishing an essential link. In the CARA-normal framework with the surplus allocated according to the Myerson value and all agents being homogeneous, a pair forming an essential link received the full social surplus created by the link. This implies that the social and private benefits coincide in the benchmark model for essential links, and therefore there is no within group underinvestment for any cost of link formation. For general utility functions and surplus allocation rules such equivalence does not hold, therefore no within group underinvestment cannot be expected to hold for all possible costs of link formation. However, for any specification of the general model that satisfies the assumptions above (in particular that the private benefit of establishing any link is always strictly positive), there is no within group underinvestment *if* the cost of establishing a link between agents from the same group is small enough. While this is a nontrivial assumption, it is realistic in many settings. Indeed, in the data we consider, within group underinvestment does not appear to be a problem.

Assumption 16 (Within Group Cost of Link-formation Small)**.** *For all networks L,*

$$c_w/2 < \min_{L,i,j \ st. \ C_i(L) \neq C_j(L)} u_i^{\tau}(L \cup \{l_{ij}\}) - u_i^{\tau}(L).$$

Assumption 16 immediately implies that if all agents are from the same group then in all stable networks there is a single component. The next proposition shows that the same holds for all efficient networks. For the rest of the section, Assumptions 12-16 are maintained.

A network is Pareto efficient if there is a feasible transfer agreement that could be reached on that network such that there is no other network, feasible transfer agreement pair in which all agents are weakly better off and some agents are strictly better off.

Proposition 17. *If all agents are from the same group then a network is Pareto efficient if and only if it is a tree connecting all agents.*

Proof. First, we consider the "only if" direction. In any Pareto efficient network, every component has to be a tree. This is because if any component was not a tree then a link could be deleted and the same risk-sharing arrangement can be achieved as before, but the costs of establishing the link saved. Now suppose there are two components of a Pareto efficient network L that are not connected. Let agents i and j be on different components. By Assumption 16, total expected utilities (that is, taking into account the costs of network formation, too) of both i and j are strictly higher for network $L \cup \{l_{ij}\}$ than for network L, while by Assumption 13 all other agents' total expected utilities are weakly higher for $L \cup \{l_{ij}\}$ than for L. This contradicts that L is Pareto efficient.

We now consider the "if" direction. Consider a tree network and suppose we implement a risk sharing agreement in which $c_i(\omega) = c_j(\omega)$, for all i and j and all states ω. As all agents' consumptions are equalized in all states, there is then no way in which link formation costs can be redistributed and the risk sharing arrangement changed, without making someone worse off. Suppose, towards a contradiction, that we can redistribute the link formation costs, by forming a different tree network, and find a new feasible consumptions that together constitute a Pareto improvement. Holding consumptions fixed, the change in network will make some agents worse off if any agents are made better off. Thus, to achieve a Pareto improvement, consumptions will have to be changed. Let $c'(\omega)$ be the new consumption vector. As the utility function $v(\cdot)$ is concave, Jensen's inequality implies that

$$\frac{1}{n}\sum_i v(c_i'(\omega)) < v\left(\frac{1}{n}\sum_i c_i'(\omega)\right) = \frac{1}{n}\sum_i v(c_i(\omega)),$$

for all ω. Thus the average expected utility from consumption will decrease. As total link formation costs have remained constant, this implies that at least one agent must be worse off. This is a contradiction. □

Corollary 18. *When all agents are from the same group, there is no underinvestment.*

Given Proposition 17, Corollary 18 follows immediately from Assumption 16 and we omit a proof.

Note that for any non-essential link $|C_i(L)| = |C_i(L) \cup \{l_{ij}\})|$. Thus the marginal benefits from i and j forming a superfluous links depend only on the distance between i and j on L and the number of agents in their component. The latter is n for any efficient network, by Proposition 17. Thus the marginal benefit from i and j receive from forming a superfluous link depend only on the distance between i and j, and are increasing in this distance. Thus an efficient network will be stable if and only the maximum distance between any two agents is sufficiently low. The next Corollary formally states this result.

Corollary 19. *If all agents are from the same group then an efficient network is stable if and only if its diameter is sufficiently small.*

Proof. Consider an efficient network L. As L is efficient there exists a unique path between i and j for all i and all $j \neq i$. Consider two such agents i and $j \neq i$. Assumption 14 implies that $d(i, j, L)$ is strictly increasing in the path length between i and j, and that $d(i, j, L) = d(j, i, L)$. Further, as $|C_i(L)| = |C_i(L \cup \{l_{ij}\})| = n$, by Assumption 15

$$u_i^\tau(L \cup \{l_{ij}\}) - u_i^\tau(L) = g(d(i, j, L), n, n) = g(d(j, i, L), n, n) = u_j^\tau(L \cup \{l_{ij}\}) - u_j^\tau(L).$$

Moreover, by Assumption 15, $g(d(i, j, L), n, n)$ is strictly increasing in $d(i, j, L)$. Thus for all i and $j \neq i$, there exists a threshold $\hat{d}$ such that i and j benefit from forming a superfluous link if and only $d(i, j, L) > \hat{d}$.

As there is never any underinvestment by Corollary 18, no agent can benefit from deleting a link on L. Thus the network L is stable if and only if no two agents can benefit from forming a superfluous link. Hence L is stable if and only if $\max_{i,j} d(i, j, L) \leq \overline{d}$. As $d(i, j, L)$ is strictly increasing in the (unique) path length between i and j, this is equivalent to the diameter of L being sufficiently small. □

A network is *least* stable within a class of networks, when its stability implies the stability of any other network in that class. A network is *most* stable within a class of networks, when its instability implies the instability of any other network in that class.

Proposition 20. *If all agents are from the same group then*

(i) *the most stable efficient network is the star,*
(ii) *the least stable efficient network is the line.*

Proof. By Corollary 19 an efficient network is stable if and only if its diameter is sufficiently low. It follows that if a network with diameter d is stable, all efficient networks with weakly lower diameter will also be stable. As the line network maximizes diameter among efficient networks, its stability implies the stability of all other efficient networks and it is least stable. Similarly, if a network with diameter d is unstable, Corollary 19 implies that all network with a weakly higher diameter are unstable. As the star network minimizes the diameter within the class of efficient networks, its instability implies the instability of all other efficient networks, and it is most stable within the class of efficient networks. □

Inequality measures within the Atkinson class will often rank utility vectors differently. In the simpler setting, with CARA utilities, normally distributed incomes and the Myerson value allocation rule we were able to identify the star as the least equitable networks for any inequality measure in the Atkinson class. This was achieved by showing that any efficient network could be transformed into a star by rewiring it in a way such that, at each step of the rewiring, the utility of the center agent increased, the utility of one other agent decreased and the utility of the remaining agents remained constant. Specifically, the act of removing a link l_{ij} and adding a link l_{jk}, increased the utility of agent k, decreased the utility of agent i and held constant the utility of all other agents.

In the more general setting, this rewiring need not hold constant the utility of the other agents. This creates problems. Consider the four agent line network and suppose utilities, after link formation costs, are $(10, 25, 25, 10)$. Now suppose we remove link l_{34} and add link l_{24} to create a star network. In the more general model, utilities after this rewiring might be $(11, 35, 11, 11)$. These two vectors will be ranked differently by different inequality measures within the Atkinson class. However, if we make an additional assumption that this kind of rewiring only affects those agents who gain or lose a link, then we can relate inequality to network structure in the more general setting.

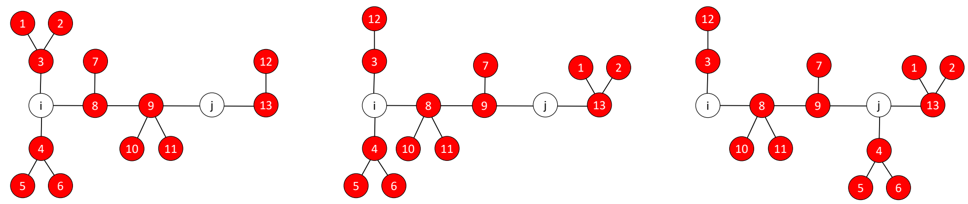
Proposition 21. *Suppose there is one group, and for all pairs of efficient networks L and L' such that $L' = \{L \setminus l_{ij}\} \cup l_{jk}$, the transfer arrangements satisfy $\tau_l(L) = \tau_l(L')$ for all $l \neq i, k$. Then for all inequality measures in the Atkinson class, among the set of efficient network, star networks and only star networks maximize inequality, while line networks and only line networks minimize inequality.*

Proof. We begin with a Lemma:

Lemma 22. *Suppose there is one group, and for all pairs of efficient networks L and L' such that $L' = \{L \setminus l_{ij}\} \cup l_{jk}$, the transfer arrangements satisfy $\tau_l(L) = \tau_l(L')$ for all $l \neq i, k$. Then agents with a higher degree in L have a higher utility.*

Proof. Consider an efficient network L and suppose agent i has higher degree than j. We will show that we can rewire a network in a way that weakly reduces i's utility and increases j's utility, but swaps the positions of i and j in the network such that on this new network i should have the same utility j had on the initial network. This will imply that i must have had a higher utility on the initial network.

Consider the following rewiring, an example of which is illustrated in Figure 5. As L is efficient it is a tree by Proposition 17 and there is a unique path between i and j. If i is directly connected to j we do not need to do any rewiring along this path. Otherwise, let



(A) Initial Network L

(B) L'

(C) Rewired network L''

FIGURE 5. An example of the rewiring used to find a contradiction in the proof of Lemma 22 is shown. Panel (i) shows the initial network, Panel (ii) the interim network and Panel (iii) the final network after the rewiring is complete.

there be $l \geq 1$ agents on this path, other than i and j, and create the following two labelings of these agents: $i, i1, \ldots, il, j$ and $i, jl, \ldots, j1, j$. Thus $i1 = jl$, $i2 = j(l-1)$, and so on. Now, if agent $i1$ has a link to an agent k on L, and k is not on the path between i and j, we remove the link $l_{i1,k}$ and add the link $l_{j1,k}$. Repeat until all of $i1$'s links to agents not on the shortest path between i and j have been rewired. We now repeat for ik, with $k = 2, \ldots l$. Note that at each step of this rewiring we reach a connected tree network.

Consider now the neighbors of j not on the path between i and j. Match each of these neighbors to a different neighbor of i's who is also not on this path. As i has a higher degree than j, such a matching exists. For each such pair we start with j's neighbor. Letting this neighbor of j be k, one by one, we rewire each of k's links on L, except l_{ij}, to the neighbor of i agent k was matched to. Let this agent be l. We then rewire each of l's links on L, except l_{il}, to agent k. Repeat for all of j's neighbors on L not on the path between i and j. Note again that at each step of this rewiring we reach a tree network. After all this rewiring, let the network that has been reached be denoted L'.

As in all the rewiring so far i and j have kept the same links, and as at each step an efficient network has been reached, by the premise of the Proposition, $\tau_i(L) = \tau_i(L')$ and $\tau_j(L) = \tau_j(L')$, so $u_i^\tau(L) = u_i^\tau(L')$ and $u_j^\tau(L) = u_j^\tau(L')$.

Finally, we consider the neighbors of i who were not on the shortest path to j, and were not matched to one of j's neighbors. As i's degree is higher than j's there exists at least one such agent. For all agents in this set, we remove their link to i and add a link to j. Let the network reached after this be denoted L''.

By Assumption 12, this increases j's utility and decreases i's utility, so $u_i^\tau(L) = u_i^\tau(L') > u_i^\tau(L'')$ and $u_j^\tau(L) = u_j^\tau(L') < u_j^\tau(L'')$. However, by construction, after this rewiring is complete i's position in L'' is identical to j's position in L (up to a relabeling of agents), while j's position in L'' is identical to i's position in L. Thus by Assumption 15 $u_i^\tau(L) = u_i^\tau(L'')$ and $u_j^\tau(L) = u_j^\tau(L'')$. We then have that

$$u_i^\tau(L) = u_i^\tau(L') > u_i^\tau(L'') = u_j^\tau(L).$$

□

We can now prove the Proposition. As shown in the proof of Proposition 6, the star network can be reached from any efficient network L by rewiring links to the highest degree agent in L. By Lemma 22 the agent with the highest utility on L is the agent with the highest degree, and by Assumption 16, the net expected utility of this agent increases at each such step of the rewiring, while the net expected utility of all other agents weakly decreases. The argument from the proof of Proposition 6 can then be applied again, and utilities become more unequal for any inequality measure in the Atkinson class.

The argument for the line network is equivalent. From any efficient network L, there is a rewiring to the line network that decreases the utility of the highest degree agent at each step, which by Lemma 22 is also the highest utility agent, and increases the utility of all other agents. Thus, utilities become more equal for any inequality measure in the Atkinson class. □

We will now consider the multiple group case. With one group it was efficient for a network to form in which all agents are path-connected to each other. We now make an assumption to ensure this remains the case with multiple groups.

Assumption 23 (Efficient Risk Sharing Across Group)**.** *For any network L with at least two components there exists a risk sharing agreement τ, and a pair of agents i and $j \notin C_i$, such that all agents are weakly better off on $L \cup \{l_{ij}\}$ and some agents are strictly better off.*

Relative to the single group case, agents from different groups provide each other with access to less correlated income streams. This increases the total surplus generated by risk sharing conditional on a given network being formed. Moreover, the presence of across group links provides positive externalities to others insofar as it increases the marginal value of within group links. This raises the question of how the additional surplus generated by across group risk sharing should be split among the agents. We take a parsimonious approach to this issue by making two assumptions. The first assumption builds on the single group analysis. It requires that agents receive at least the same marginal benefits they would receive were all agents from the same group. The additional surplus generated must be split in a way such that each agent receives a weakly positive share.

Assumption 24 (Lower Bound)**.** *Consider a network L, such that l_{ij} is essential on $L \cup \{l_{ij}\}$, and two allocations of the agents to groups G, G'. If all agents are from the same group under G, such that $G(i) = G(j)$ for all $i \neq j$, then*

$$u_i^\tau(L \cup \{l_{ij}\}, G') - u_i^\tau(L, G') \geq u_i^\tau(L \cup \{l_{ij}\}, G) - u_i^\tau(L, G).$$

This assumption requires that the additional benefits an agent i gets from risk sharing, in terms of the second period agreement reached relative to the payoff i would have got were everyone from the same group, strictly increase if a link l_{jk} is removed form the network and replaced by a link l_{ij} without changing the set of agents in each component.

Assumption 25 (Link Increasing Additional Benefits)**.** *Consider two networks L and L' connecting the same sets of agents, and two allocations of the agents to groups G, G'. If L' can be reached from L by rewiring a link to i such that, $L' = \{L \setminus l_{jk}\} \cup l_{ij}$, $i \neq j \neq k$, $l_{ij} \notin L$, $l_{jk} \in L$, G' contains agents from different groups and under G all agents are from the same group, then*

$$u_i^\tau(L', G') - u_i^\tau(L, G') > u_i^\tau(L', G) - u_i^\tau(L, G).$$

Assumption 25 is only a coarse partial ordering on utilities. While it implies that an agent's share of the additional surplus generated by across group risk sharing increases as links are rewired to that agent, it makes no comparison between networks that cannot be reached by rewiring links to a single agent. In particular, following a rewiring to i, it does not pin down how the payoffs of other agents changes.

Proposition 26. *Suppose all groups have the same utility functions, such that $v_i = v_j$ for all i, j. With k different groups, there exist a $\bar{\kappa}_W > 0$ such that for all $\kappa_W < \bar{\kappa}_W$ a network is Pareto efficient if and only if it is a tree with $k-1$ across group links.*

Proof. We begin by showing the "only if" direction. All Pareto efficient networks are trees. First, by Assumption 23, risk sharing among all agents is efficient so L must connect all agents. Second, a Pareto improvement can be achieved on any connected non-tree network by implementing the same risk sharing arrangement and deleting a superfluous link, thereby saving these costs.

We now show that efficient networks must also have exactly $k-1$ across group links. We will show, by construction, that for any tree network with strictly more than $k-1$ across group links, there exists a Pareto improvement.

If there are more than $k-1$ across group links in a tree network, we claim that there must exist an across-group link l_{ij} which, upon its removal, will result in a network $L' = L \setminus \{l_{ij}\}$ such that there exists two agents (k, l), with $G(k) = G(l)$ and $C_k(L') \neq C_l(L')$.

Towards a contradiction, let there be $k' > k-1$ across group links and suppose this is not true. As L is a tree network, removing all across group links must then result in there being $k'+1$ components. If there are no agents from the same group in different components, this implies that there must be at least $k'+1 > k$ different groups, which would be a contradiction. Thus there exist two components each containing an agent from the same group. Denote these agents k, l. As L is a tree there exists a unique path between k and l on L, and as k and l are in different components following the removal of across group links there exists at least one across group link on this path. Letting this link be l_{ij} proves the claim.

As k, l are in different components on L', but from the same group, the network $L'' = L' \cup l_{kl}$ will be a connected tree network with one less across-group link, and one more within-group link than L.

On the network L'' we implement the same risk-sharing arrangement as before, with one exception. First we identify the vector of consumptions for agents i and j that make them just as well off as on the original network, and continue to satisfy the Borch rule:

$$\frac{\partial v_i(c_i(\omega))/\partial c_i(\omega)}{\partial v_i(c_i(\omega'))/\partial c_i(\omega')} = \frac{\partial v_j(c_j(\omega))/\partial c_j(\omega)}{\partial v_j(c_j(\omega'))/\partial c_j(\omega')} = \frac{\partial v_{i'}(c_{i'}(\omega))/\partial c_{i'}(\omega)}{\partial v_{i'}(c_{i'}(\omega'))/\partial c_{i'}(\omega')},$$

for all states ω, ω' and all $i' \neq k, l$.

As i and j save the cost of an across group link, and utility is strictly increasing and concave in consumption, this implies that $c_i(\omega)$ and $c_j(\omega)$ must strictly decreases in all states ω. This additional consumption is passed onto agents k and l. As there is a strictly positive amount of remaining consumption in all states of the world, and utilities are strictly increasing in consumption, there exist feasible consumption vectors for agents k and l that strictly increase $\mathrm{E}(v(c_k))$ and $\mathrm{E}(v(c_l))$. Thus, for all κ_w sufficiently small, we have $\mathrm{E}(v(c_k)) > \kappa_w$ and $\mathrm{E}(v(c_l)) > \kappa_w$. We have therefore constructed a Pareto improvement.

We now show the "if" direction. Consider a tree network with $k-1$ across group links. Suppose we implement a risk sharing agreement in which $c_i(\omega) = c_j(\omega)$, for all i and j. As all agents' consumptions are equalized in all states, there is then no way in which link formation costs can be redistributed and the risk sharing arrangement changed, without making someone worse. Suppose towards a contradictions that we can redistribute the link formation costs, by forming a different tree network with $k-1$ across group links, to generate a Pareto improvement. Holding consumption fixed, on the new network if some agents are better off, then some will be worse off. Thus, to achieve a Pareto improvement, consumptions will have to be changed. Let $c'(\omega)$ be the new consumption vector. As the utility function $v(\cdot)$ is concave, Jensen's inequality implies that

$$\frac{1}{n}\sum_i v(c'_i(\omega)) < v(\frac{1}{n}\sum_i c'_i(\omega)) = \frac{1}{n}\sum_i v(c_i(\omega)),$$

for all ω. Thus the average expected utility from consumption will decrease, and total link formation costs have remained constant, so at least one agent must be worse off. This is a contradiction.

□

In our simple CARA utility, normally distributed incomes, Myerson value allocation rule model, underinvestment across group is possible but there is no underinvestment within group. The same example establishes the possibility of underinvestment across group in our more general setting. There is also never any underinvestment within group in our more general setting as we now show.

Proposition 27. *There is never any underinvestment within group.*

Proof. Consider any stable network L' and allocation to groups G'. Suppose, towards a contradiction, there is underinvestment within a group in L'. There must then be an essential link l_{ij} the planner could form to achieve a Pareto improvement. Stability of L' implies that either $u_i^\tau(L' \cup \{l_{ij}\}, G') - u_i^\tau(L', G') < c_w$ or else $u_j^\tau(L' \cup \{l_{ij}\}, G') - u_j^\tau(L', G') < c_w$. Without loss of generality suppose $u_i^\tau(L' \cup \{l_{ij}\}, G') - u_i^\tau(L', G') < c_w$. Consider now the alternative grouping G in which all agents are from the same group. In this case, by Assumption 16 and as l_{ij} is essential, $u_i^\tau(L' \cup \{l_{ij}\}, G) - u_i^\tau(L', G) \geq c_w$. Thus, combining inequalities, $u_i^\tau(L' \cup \{l_{ij}\}, G) - u_i^\tau(L', G) > u_i^\tau(L' \cup \{l_{ij}\}, G') - u_i^\tau(L', G')$. This contradicts Assumption 24. □

Consider the partial ordering in which an agent i is more central in a network L' than in network L if and only if L' can be reached from L by rewiring links only to i. The following result generalizes the result in the benchmark model that more centrally located agents within a group have higher incentive to create across group links.

Proposition 28. *Suppose that*

(i) *when there is one group, for all efficient networks $L \cup \{l_{ij}\}$, $g(\bar{d}, |C_i(L)|, |C_i(L \cup \{l_{ij}\})|) = g(\bar{d}, |C_j(L)|, |C_j(L \cup \{l_{ij}\})|)$; and*

(ii) *there are two groups.*

Then, for any efficient network L with across group link l_{ij}, if it is profitable for an agent i to form l_{ij}, and the alternative efficient network L' can be reached from L by rewiring within group links to i, then it is also profitable for i to form the link $l_{ij} \in L'$.

Proof. Let G' be the grouping of agents. Agent i is weakly better incentivized to invest in the across group link l_{ij} on the network L' than the network L if and only if

$$u_i^\tau(L, G') - u_i^\tau(L \setminus \{l_{ij}\}, G') \leq u_i^\tau(L', G') - u_i^\tau(L' \setminus \{l_{ij}\}, G'). \tag{35}$$

As L and L' are efficient, and l_{ij} is an across group link on both L and L', all agents who are path-connected to i on $L \setminus \{l_{ij}\}$ are from the same group as i, as are all agents path connected to i on $L' \setminus \{l_{ij}\}$. Thus, on the networks $L' \setminus \{l_{ij}\}$ and $L \setminus \{l_{ij}\}$, by Assumption 24 agent i must then get exactly the same payoffs as he would do in the one group case: $u_i^\tau(L \setminus \{l_{ij}\}, G') = u_i^\tau(L \setminus \{l_{ij}\}, G)$ and $u_i^\tau(L' \setminus \{l_{ij}\}, G') = u_i^\tau(L' \setminus \{l_{ij}\}, G)$, where G is the grouping in which all agents are from the same group. We can therefore rewrite equation 35 as

$$\begin{aligned} u_i^\tau(L, G') - u_i^\tau(L, G) + u_i^\tau(L, G) - u_i^\tau(L \setminus \{l_{ij}\}, G) \quad \leq \quad & u_i^\tau(L', G') - u_i^\tau(L', G) \\ & + u_i^\tau(L', G) - u_i^\tau(L' \setminus \{l_{ij}\}, G). \end{aligned} \tag{36}$$

Repeatedly applying Assumption 25, $u_i^\tau(L,G') - u_i^\tau(L,G) < u_i^\tau(L',G') - u_i^\tau(L',G)$. Thus a sufficient condition for equation 36 to hold is that:

$$u_i^\tau(L,G) - u_i^\tau(L \setminus \{l_{ij}\},G) \leq u_i^\tau(L',G) - u_i^\tau(L' \setminus \{l_{ij}\},G).$$

As we are in the one group case and l_{ij} is essential on both L and L', $u_i^\tau(L,G) - u_i^\tau(L \setminus \{l_{ij}\},G) = u_i^\tau(L',G) - u_i^\tau(L' \setminus \{l_{ij}\},G) = g(\overline{d})$. This completes the proof.

□

Appendix B. Over and Under Investment Examples

In this Section we provide an example of over-investment within group in the unique stable network and a related example of underinvestment across group in the unique stable network.

We begin by assuming there is one group with s members connected by a network L. Equation 11 implies that Myerson distance of two agents i,j such that $l_{ij} \notin L$ is greater than $1/2$, while the Myerson distanace between i and j if they form the link l_{ij} would be $1/2$. Thus i and j's gross payoff strictly increases if the link l_{ij}. So, for κ_w sufficiently close to 0, in all stable networks for any pair of agents i,j the link l_{ij} must be formed; The unique stable network is the complete network and there is overinvestment.

Suppose now there two groups, g, g' both with s members and keep the same parameter values from the previous example. By equation 13 the incentives for form within group links are weakly increased by any across group links. Thus in all stable networks the network structures within-group must be complete networks; All possible within-group links must be formed. Suppose these are the only links formed so that no across-group links are formed. Denote this network L. From equation 15 the change in total variance achieved by connecting an agent i from group g to an agent j from group g' is strictly increasing in the size of both groups s. Considering the Myerson value calculation (equation 6), this means that the marginal contribution of the link l_{ij} to total surplus (the certainty equivalent value of the variance reduction) is strictly greater on $L \cup \{l_{ij}\}$ than it is on any strict subgraph, including all those formed when the later of i and j arrives in the Myerson calculation. This implies that $(MV(i;L \cup l_{ij}) - MV(i;L)) + (MV(j;L \cup l_{ij}) - MV(j;L)) < TS(L \cup l_{ij}) - TS(L)$ for all $l_{ij} : i \in \mathbf{S}_g, j \in \mathbf{S}_{g'}$. So, setting κ_a such that

$$MV(i;L \cup l_{ij}) - MV(i;L) + MV(j;L \cup l_{ij}) - MV(j;L) < 2\kappa_a < TS(L \cup l_{ij}) - TS(L),$$

the network L is the unique stable network and there is underinvestment (in across-group links) in all stable networks.