Procedural cum Endstate Justice:

An Implementation Viewpoint*

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Overview of the Paper

In Section 1 we describe a general model of mechanism for the allocation of resources that encompasses most of the recent interface between the social choice and implementation literatures.

We argue in Section 2 that this model allows to explore the dichotomy between endstate justice and procedural justice. Section 3 argues that no doctrine of distributive justice can rely exclusively on one form, endstate or procedural: this is a rendering of the familiar "means matter as well as ends" in the context of our formal model.

Section 4 presents the main argument of the paper, that the properties of strategyproofness and groupstrategyproofness provide a natural bridge between the two approaches to distributive justice, by making them formally indistinguishable.

In Section 5 we review the literature on strategyproofness in the last twenty-five years or so. We uncover two chronological phases in the research effort: a first phase until 1979 produced mostly "negative" results by working in very general environments, whereas a second phase starting in 1980 offers more "positive" results by looking at more structured microeconomic environments.

1. *A General Model of Distributive Justice*

Distributive justice concerns the processes by which material resources are allocated among the members of a given society (hereafter called the "agents"). Two aspects of these processes have been at the forefront of the normative discussion ever since the Enlightenment: *i*) with what *rights* are individual agents endowed, what influence can they have on the final allocation, by what free, unconstrained actions; *ii*) to what degree does the actual distribution of resources, the outcome, or *endstate,* fulfill the individual values and aspirations of the agents; what level of *welfare* does it allow them to reach?

The conceptual dichotomy between rights and welfare has deep roots in political philosophy: a good example is Berlin's richly nuanced distinction between negative freedom: the extent of my rights, and positive freedom: the freedom of choosing my own goals and aspirations, and my ability to fulfil them; roughly, the choice of my own welfare ordering and my ability to reach a high level of welfare. See Berlin [1969]. In the last two or three decades a substantial research effort in microeconomic theory, at the juncture of the social choice and the mechanism design literatures, has developed a systematic formal analysis of these two components of distributive justice and of their interplay in a model where the main exogenous ingredients are the distribution of individual rights and the evaluation of individual welfares.

We now describe the simplest version of a general microeconomic model of distributive justice that will suffice, despite its vagueness, to the subsequent discussion.

- the "society" is the set $N=\{1,2,...,n\}$, with generic agent $i, i\in N$.
- the set Ξ of feasible states of the world, or endstates; in the standard models an endstate is an allocation of certain commodities, private goods, public goods, etc., among the agents, so that Ξ embodies the set of technological constraints.
- the set $\Re$ of possible "welfares"; where each particular element R is defined over the set of feasible states of the world and takes into account all the relevant idiosyncratic components of the welfare of a certain agent, such as preferences, e.g., values, aspirations, but also tastes and needs, such as utilities if cardinal utilities are deemed relevant, such as endowments of

talents and skills, disabilities, etc. Note that we use the familiar notation for a preference ordering because most of the literature reviewed in this paper use no other relevant individual characteristics.

- the game form G describing the allocation of rights, and consisting of a set M_i of actions among which agent i freely chooses one element m_i, and a mapping f from $M_1 x ... x M_n$ into Ξ, specifying which endstate z will result for each profile $(m_1,...,m_n)$ of individual actions.[1]

Concrete examples of the general model $(N, \Xi, \Re, G)$ are given below. It is important to stress the most heroic assumptions on which the model rests. The three basic components, welfare (R_i), rights (M_i) and the set of endstates (Ξ) are three separate entities whose interaction is limited to two aspects. First, the welfare R_i is predicated on actions (m_i) and endstate (z); that is, individual actions and the endstate determine the welfare level. Second, the endstate (z) depends on the profile of actions (m_i) via the mapping f.

Conspicuously absent in the model are two important elements of the interaction of rights and welfare. First, the fact that the nature and extent of my rights affect my welfare: which game I play, and what role I have in it matters to me. Formally the *whole set* M_i influences R_i not just the particular action m_i I take. Second, I bear some responsibility in shaping my own welfare as conveyed in the basic liberal tenet: I am free to choose my values and goals. Therefore my actions m_i influence the *shape of* R_i, they are more than one input in the measurement of a given welfare relation R_i. These two points are abundantly discussed by the literature on distributive justice: the whole debate about "primary goods" (Rawls [1971]) and capabilities (Sen [1985]) is motivated by the latter point: agents who adapt their expectations to their lot enjoy a higher welfare than those who do not, and this responsibility should be accounted for when we try to select a fair endstate. See also the literature on responsibility by Fleurbaey [1995] and Roemer [1993]. On the first point, there is a substantial literature on extending ordinary welfare relations to opportunity sets, e.g., Pattanaik and Xu [1990].

Another simplifying assumption of the model is the identification of the game form *G* with an allocation of rights. Since Gärdenfors [1981], this is an approach endorsed by many authors, e.g., Gaertner, Pattanaik, and Suzumura [1992]. A more traditional approach insists that a right confers on an individual the ability to fix some features of the final outcome (see, e.g., Sen [1970]: that the two approaches are not identical is discussed in particular by Deb, Pattanaik, and Razzolini [1997].

2. *Endstate Justice versus Procedural Justice*

These shortcomings notwithstanding, the logical separation of welfare and rights has considerable advantages. In a general microeconomic model like the one just described, the separation of ends, the distribution of welfare and means, the set of actions and the game form G, is especially convenient to describe formally and analyze two familiar yet one-sided doctrines of social justice. One doctrine is exclusively concerned with endstate justice, and the other exclusively concerned with procedural justice. The former doctrine can be traced back to classical utilitarianism, and the latter to the libertarian tradition. Without attempting to elucidate the intellectual ancestry of the two concepts, we will define endstate and procedural justice in the context of our general model.

Endstate justice is a judgment on a profile $(R_1,...R_n)$ of individual characteristics and a state of the world z; it tells us whether this particular endstate is equitable at this particular profile. The test for calling an endstate equitable depends on the distribution of individual welfare relations, or, more generally, of any relevant individual characteristics with which the agent is passively endowed. For the usual reasons of dispersion of the relevant (private) information between the individual agents and incentive compatibility, it will be necessary to use an appropriate game form G to *implement* the equity criterion that we select. "We" here refers to the benevolent dictator, or assembly of citizens in the original position who chooses and enforces G. However, neither the way the agents will play the game G, i.e., the particular actions they choose, nor the genuine equity of the game form itself, whether or not it is a fair game to play, matter. The judgment of justice is entirely ex post, oblivious to the means by which the endstate was reached.

Note that endstate justice as defined here is a generalized version of welfarism in the sense that it uses criteria of justice that go beyond the mere comparison of welfare, or even primary goods, among agents. These criteria may give a direct role to voluntary characteristics as in cost sharing: how to share total cost given a profile of individual demands. They may compare shares across agents as in the no envy test or even check that the endstate responds correctly to changes in the data of the problem as in solidarity properties. See, e.g., Moulin and Thomson [1995]).

Procedural justice is a judgment on the game form itself, independent of the way the game is subsequently played by the agents, hence independent of the particular endstate reached. For procedural justice, the rules of the game, the procedure, embody the entirety of the concerns for equity. No outcome (endstate) can be deemed unfair provided the rules of the game have been scrupulously observed in the path leading to it, just like a fair lottery cannot produce an unfair outcome. The judgment of justice is entirely ex ante, and bears only on the means regardless of the ends.

Endstate justice and procedural justice as objects of normative analysis pursue similar goals by similar axiomatic means. They seek to select a solution to a given problem of resource allocation by means of a suitable set of axioms (desirable properties). Of course this solution is a distribution of rights, a game form, in the case of procedural justice, and a social choice function, associating to every profile of characteristics a certain endstate deemed just, in the case of endstate justice. For a general discussion of the axiomatic method, see Moulin and Thomson [1995].

Yet despite the normative analogy, endstate justice and procedural justice are two clearly distinct endeavors, at least in the model, and the exclusive view of social justice that each of them idealizes is meaningful, both logically— formally coherent—and intuitively—it matches the casual usage of the terms fair play and fair outcome.

3. *The Untenability of One-sided Endstate or Procedural Justice*

Insisting that means matter as well as ends is commonplace: a clear formulation is, for instance, Sen [1992].

In a microeconomic model like the one in Section 1, we can easily find examples where a fair procedure yields an unfair endstate, or a fair endstate results from an unfair procedure. Examples of the former include provision of a public good by voluntary subscription, where often a single agent ends up bearing the entire cost of the good. See, e.g., Moulin [1995], Chapter 6. In the celebrated “gloves market” with 101 owners of a right glove and 100 owners of a left glove the unique core allocation gives all the surplus to the left glove owners. Examples of the latter include the choice of the “just” candidate by a dictator, or of an envy-free division of the cake by the Divide and Choose method— deemed unfair because it endows the Divider with a strategic advantage.

The above examples do not disturb the fanatic of procedural justice (resp. of endstate justice), because he is precisely making the point that arguments of endstate (resp. procedural) justice should not be considered *even* when they are begging for our ethical attention. We will now invoke more powerful arguments against our two one-sided views, starting with procedural justice.

The strict interpretation of procedural justice is not concerned with the strategic properties of the procedure. Whether or not the game has zero, many, or a single equilibrium outcome does not matter. Even irrational actions by the agents are of no concern, e.g., my negative freedom includes the privilege of being unpredictable. See Berlin [1969]. Yet this extremist position is refuted by the normative concern for efficiency , Pareto optimality. Pareto optimality is the only test of collective rationality of the endstate that is common to all doctrines of the social order derived from methodological individualism. The argument is as follows: if we like our procedure, i.e., game form, to bring about an efficient endstate, then we must pay attention to its strategic properties. In particular,

- a game form with no equilibrium for some specifications of individual characteristics does not allow for the rational behavior of the agents. Because the notion of rational behavior is formalized by the equilibrium concept anything can happen, including inefficiencies. A typical example is majority voting when the majority relation has a cycle. Where we stop in the cycle may be a matter of pure luck or of superior political skills by some participants. However the cycles of the majority relation typically encompass large subsets of inefficient outcomes so there is the very real possibility of an inefficient endstate. This point is emphatically demonstrated by McKelvey [1976] in the spatial voting context.

- a game form with a large set of equilibrium outcomes, e.g., dividing a surplus by means of the Nash demand game, (Nash [1950]) leads to severe coordination problems in selecting a particular equilibrium outcome, thus is conducive to inefficiencies as well. The standard configuration is that of a "battle of the sexes," which results in the inefficient outcome if several agents commit themselves to incompatible demands.

To meet the minimal normative requirement of efficiency under pure procedural justice, a necessary property is that the game form yields a unique strategic equilibrium outcome; moreover this outcome must pass the test of Pareto optimality. Here the equilibrium concept, or scenario of rationality, must be chosen to fit the particular circumstances of the particular allocation problem at hand.

A good example where these requirements are met, is the exchange and distribution of private goods under the assumptions of a competitive environment (to wit, many agents with convex preferences and a "small" initial endowment relative to the market size): there the set of competitive equilibrium allocations, also interpretable as the core, is small and efficient so the invisible hand "works."

On the other hand, many procedures commonly used to allocate certain scarce resources, display either a unique but inefficient equilibrium endstate, or a large or empty set of such equilibria. Examples of the former include the exploitation of common property resources by free access, namely, the tragedy of the commons, Hardin [1968]; and the provision of public goods by voluntary subscription the classic exposition of which is by Olson [1965]. Examples of the latter include voting by a qualified majority: depending on the quota, the size of the qualified majority, the set of equilibrium outcomes in the sense of the core, oscillates between the empty set if its quota is low, and a large set if it is high. See Nakamura [1979]. The underlying reason for these difficulties is the widespread role of externalities, which is miraculously absent in the case of private exchange of private goods.

The upshot of the strategic analysis of game forms is that pure procedural justice does not provide a *natural* system of rights in many important allocation problems. If private ownership is *natural* in the context of exchanging private goods because the resulting game of exchange has the desirable strategic feature, we do not have a *natural* system of rights, a canonical game form, to offer as a solution to such important questions as:

- what quota should be required to amend the constitution? what voting rule should be followed in electing a president?
- how we should alter the rights of free access to exhaustible resources such as the sea or the wilderness to avoid the tragedy of the commons. Ostrom [1991] comments at length on the idiosyncratic character of observed practical answers.
- how we should produce and trade under decreasing marginal costs? how to regulate natural monopolies?
- in general, how should rights be allocated when externalities are widespread, e.g., in the matter of protecting the environment?

Therefore, the justification of a particular game form in a particular problem must use arguments based on the resulting endstates under a given equilibrium concept and so the ends must matter, after all.

We now turn to the objections to pure endstate justice. These objections hinge upon the resolution of the implementation problem alluded to in Section 2. Some of the individual characteristics included in the parameter R_i are privately known only to the concerned agent and perhaps to some of her fellow citizens, hence the benevolent dictator or mechanism designer must use a game form to elicit these private characteristics, either directly by asking from each agent that he or she simply reports these variables, or indirectly by designing a game form of which the equilibrium outcome coincides precisely with its endstate objectives of justice. Thus the game form implements the social choice function and achieves endstate justice. For classical statements of the implementation problem, the reader may consult Maskin [1985] or Myerson [1991]. Thereafter we assume, for simplicity and in line with most of the literature, that the parameter R_i represents a preference ordering , an ordinal type of welfare function, and that R_i varies within a common domain $\Re$, known to the mechanism designer, who however has no knowledge of which element in $\Re$ represents the preferences of any specific agent.

The benevolent dictator motivated by pure endstate justice has an opportunistic attitude toward the resolution of the implementation problem. She will use whatever mechanism best suits her goals of achieving a just endstate, even if this involves one or more of the following features:

- endowing agents with unequal rights, a non anonymous game form, even though the social choice function to be implemented is anonymous and treats equal agents equally
- using coercive threats to extract private information from the agents and/or to influence their strategic choices; she will, metaphorically speaking, burn the agents' feet, submit them to lie detectors, and other forms of duress

- using more subtle, yet no less objectionable, twists to force the agents to mutually reveal their private information, as in the mechanisms à la Maskin [1977]. This is an elaborate version of the familiar "unanimity at gunpoint" mechanism whereby the agents must unanimously agree in their report about the entire profile of characteristics, lest a very bad endstate is enforced
- changing the game form frequently, as dictated by circumstantial information, e.g., about the sharper strategic skills or more accurate information of certain agents thereby giving them a more active role in the procedure. Or her knowledge of the statistical distribution of characteristics, allows her to use Bayesian mechanisms; and so on.

All above features run contrary to the simple and pervasive intuition that agents do care about their rights and about the decision process. See Sen [1992]. In the case of the implementation problem, this means that the benevolent dictator should not be allowed to extract the private information held by individual agents in any way she pleases. She should not bully agents into revealing information without giving them some real influence on the process itself. Not to place any limits on the procedures that can be used affords too much power to the benevolent dictator. Similarly in the discussion of pure procedural justice, a large or empty equilibrium set gives "too much" influence to the individual skills and luck. Means matter as well as ends.

4. *Endstate Justice cum Procedural Justice*

Granted that means matter as well as ends, what is the next logical step? How can we combine endstate and procedural justice in the general model of Section 1?

A first clue to answer this question is the formal overlap of endstate and procedural justice. When the test of endstate justice selects a single endstate for every profile of individual preferences, this determines a mapping (from $\Re^N$ into Ξ) that is formally identical to a *direct* game form, namely one where an action by agent i is simply to report his preferences. Yet the formal identity does not convey any positive implications because the strategic equilibrium of the direct game form does not, in general, consist of reporting one's true preferences. This is where the second clue comes in.

Suppose we can choose the social choice function, a mapping φ from $\Re^N$ into Z, with the property that for all profiles $R=(R_1,...,R_n)$ reporting one's true preferences is a (Nash) equilibrium. Formally we denote by $(R|^i R_i^/)$ the profile equal to R except that R_i has been replaced by $R_i^/$, and we require the property of *strategyproofness*:

$$\forall i \in N,\ \forall R, R^/ \in \Re^N\text{:}\ \varphi(R) R_i \varphi(R|^i R_i^/)$$

Note that the equilibrium property of the action "report truthfully any preferences" is that of a dominant strategy, a noncooperative rationality property significantly more demanding than the Nash equilibrium property. It delivers a particularly simple and robust way to implement our social choice function, one that is oblivious to the degree of dispersion of the information on private characteristics, i.e., knowing what the other agents' preferences are, and/or what they will report, is worthless to me.

The convenience of the direct game form and the robustness of the dominant strategy equilibrium as a rationality scenario are two positive properties, of interest even in the context of pure endstate justice. But there is more to the appeal of strategyproofness for a social choice function. As the game form and social choice functions are the same formal object, the same tests of equity apply to both. For instance a property like anonymity—equal treatment of equal reports—of the mapping φ is at the same time a statement of endstate justice and of procedural justice. Therefore endstate and procedural justice coincide, the means are justified precisely to the extent that the ends are. In Rawls' terminology, we have a situation of "perfect procedural justice." See Rawls [1971].

In point of fact, the strategyproofness property is not quite demanding enough to pass the most thorough tests of strategic determinism and predictability. The property of *group strategyproofness* strengthens strategyproofness by making it unprofitable for any subgroup or coalition of agents to jointly misreport their preferences:

$$\forall S\subseteq N\ \forall R,R'\in\Re^N\colon \left\{\forall i\in S\ \ \varphi\left(R|^S R'\right)R_i\varphi(R)\right\} \Rightarrow \left\{\forall i\in S\ \ \varphi\left(R|^S R'\right)I_i\,\varphi(R)\right\}$$

where I_i is the indifference relation associated with R_i . Thus not only is the knowledge of other agents' actions worthless to me, the possibility of cooperating with them toward playing the mechanism collectively or by forming partial alliances is also useless. The public authority needs only to collect the messages $R_1,...,R_n$ and enforce the corresponding endstate mandated by the game form. We can then rest assured that the correct endstate will be implemented, because the participants will report their preferences truthfully lest their behavior be grossly irrational.

The advantages of (group) strategyproof social choice functions or, equivalently, game forms, are normatively appealing. Indeed, following the invention of the concept more than two decades ago by Gibbard [1973], the search for such social choice functions in a variety of contexts has been active and multifarious.

In any allocation problem captured by the general model of Section 1, there exist some group strategyproof social choice functions. Two obvious instances are the *imposed* social choice functions, always choosing the same endstate irrespective of the profile of messages, and the *dictatorial* social choice functions, choosing for all profiles an endstate that maximizes the preferences of a certain agent—the dictator—fixed once and for all.[2]

Naturally the imposed social choice functions yield unbearably inefficient endstates, and the dictatorial ones violate even the mildest requirement of endstate/procedural justice. The interesting and difficult question is: exactly how far can a social choice function go toward efficiency and fairness while keeping the (group) strategyproofness property? The question is mathematically difficult and many related problems remain unsolved. Yet the literature has accumulated a large number of partial answers from which much may be learned.

5. *An Overview of the Literature on Strategyproofness*

All papers accounted for in figures 1 and 2 attempt to characterize, or give examples of, strategyproof or group strategyproof social choice functions in a particular allocation problem. Three types of results emerge:

- *positive results*: strategyproofness is compatible with both equity and efficiency. Examples include voting with single peaked preferences, house barter, and fair division under single peaked preferences
- *negative results*: strategyproofness imposes a loss of efficiency and/or equity so severe as to force us to drop one or the other requirements altogether. Examples include voting between three or more alternatives where strategyproofness is only compatible with imposed or dictatorial social choice functions, trading of multiple private goods, and, production under decreasing marginal costs

- “*neutral” results*: strategyproofness is not compatible with full efficiency; yet we can find equitable group strategyproof social choice functions for which the efficiency loss is not too severe. Examples include public decisions with monetary compensations under quasi-linear utilities, house assignment with lotteries, and, cost and surplus-sharing under increasing marginal cost.

The papers are partitioned according to five themes corresponding mainly to the type of allocation problem analyzed with the exception of the “general” papers (see below).

a) ***voting***: the pure public decision problem.

Papers differ in the restrictions they impose on the domain of preferences over alternatives: *i*) unrestricted domain, any complete and transitive preference relation, *ii*) singlepeaked domain, uni- or multi-dimensional, *iii*) Euclidean, spatial voting, *iv*) linear, von Neumann–Morgenstern preferences over lotteries, *v*) classical microeconomic preferences.

Other papers explore voting rules with multivalued outcomes, or the likelihood of manipulations.

b) ***Clarke-Groves mechanisms***: public decisions with monetary transfers and preferences quasi-linear in money.

Here the early literature pre 1979 produced general characterization results, whereas the subsequent literature offers new applications, e.g., to bilateral assignment, or bilateral trading.

c) ***General papers***

This includes results broadly applicable to the various allocation problems: exploring, *i*) the link of strategyproofness with the general implementation theory, *ii*) its link with normative properties, notably anonymity and the No Envy property, *iii*) new and general proof techniques to characterize the class of strategyproof social choice functions, and *iv*) the general consequences of the separability of preferences.

d) ***Exchange and Distribution***: the allocation of unproduced private goods.

These include: *i*) the classical framework of Arrow-Debreu economies with private ownership or with common ownership (fair division), *ii*) the case of a single nonfreely disposable commodity under single peaked preferences, *iii*) house barter, the exchange of indivisible goods without monetary transfers, *iv*) matching and marriage models, *v*) assignment of indivisible goods with lotteries.

e) ***Production***: the joint use of a common technology, in the cost or surplus sharing format.

Here we find, *i*) models with pure private goods emphasizing the role of increasing or decreasing marginal costs of production, *ii*) the provision of pure public goods, *iii*) the production of excludable public goods, i.e., nonrival private goods.

Except for a handful of "prehistoric" papers in which the intuition of the strategyproofness property, or its violation, can be found, the analysis of the property begins in 1971 with Clarke [1971]. Historic papers include on the one hand Black [1948] on majority voting under single peaked preferences, Farqharson [1969] on strategic voting, and Dummett and Farqharson [1961] who conjectured the Gibbard-Satterthwaite impossibility; and on the other hand

Samuelson [1954] on the free rider problem in the provision of a public good, and Vickrey [1961], a precursor of the literature on Clarke-Groves mechanisms.

Figures 1 and 2 partition the flow of articles in two chronological "eras." The first era ends in 1989, and is characterized by a concentration on the themes of voting (more than ⅔ of the papers) and Clarke-Groves mechanisms (1/6 of the papers). The last two years of the first era show almost no activity. The second era starts in 1990, when the research on group strategyproofness rebounds: the average yearly rate of output is around 5.3 papers in the first era, and jumps to 11.6 in the second. The relative size of the theme of voting decreases from 68% to 36% whereas that of exchange and distribution raises from 10% to 30%. The theme of production also appears and captures almost 20% of total activity.

The tone of the first era papers is overwhelmingly negative: two seminal results, one on the voting problem with unrestricted domain, the Gibbard-Satterthwaite theorem of 1973, and the other on the problem of exchanging private goods, Hurwicz 1972, convey the forceful message that no strategyproof mechanism can be reasonably fair and efficient in either one of these two problems. While the literature elaborated the negative message of these two results, it offered only a few positive or at least neutral results. In the problem of voting with single-peaked preferences, majority voting and a number of related social choice functions are (group) strategyproof, efficient and fair. Although this result expanded the reach of Black's 1948 intuition; it was somewhat generalized to the multidimensional context under separable preferences. For the two-sided matching problem, the Gale Shapley algorithm, a fair and efficient social choice function is (group) strategyproof for one side of the market, Dubins and Freedman [1981], but not for the other and the performance cannot be improved.

By far the most important family of strategyproof social choice functions uncovered in the first era is the versatile class of Clarke-Groves mechanisms that can be applied to *any* resource allocation problem where all individual preferences are quasi-linear with respect to a numeraire good (the same for every agent). The corresponding social choice functions associate to every preference profile a feasible allocation and a profile of monetary transfers. Strategyproofness holds if the transfers are chosen in such a way that the net charge to any individual agent equals the marginal social worth of her own message, plus an arbitrary term independent of her message. This simple trick yields a partial answer to the free rider problem in the provision of a public good, Samuelson [1954], as well as a general compensation method for public decision making. It can also be used in the bilateral assignment problem , a generalization of Vickrey's 1961 observation that the second price auction is strategyproof on the buyer's side, see Leonard [1983]. However, the appeal of the Clarke-Groves social choice function is limited, severely in some contexts, for two reasons: they are inefficient, and they are manipulable by coalitions, i.e., strategyproof but not group strategyproof. See Green and Laffont [1979] or Moulin [1988], chapter 8.

Turning to the papers of the second era, three pervasive qualitative features capture the main trends of the research: *i*) models are more specialized, more driven by the microeconomic modeling of specific problems of resource allocation; *ii*) (group) strategyproofness is often combined with other normative properties, in particular equity properties, and *iii*) the results are generally positive or neutral.

In the voting problem, the popular model of a multidimensional issue which is well suited to applications in mathematical politics under is being systematically investigated various separability assumptions on individual preferences. Positive or neutral results include the choice of a committee, Barbera, Gül and Stacchetti [1993] and the Euclidean preferences model, Peters, Van der Stel, and Storcken [1993].

The simple problem of fairly dividing a single commodity under singlepeaked preferences with no free disposal yields a rich family of (group) strategyproof social choice functions, interpretable as rationing methods. If the general family is complex, Barbera, Jackson and Neme [1995], one member of the family, the uniform rationing method, stands out as the unique group strategyproof method that is also fair, Sprumont [1991]. This positive result proves helpful in the more general and as yet uncompleted search for (group) strategyproof methods allocating private goods, Barbera and Jackson [1995].

Much new ground has been broken in the models of cooperative productions where a group of users "share" a given technology in the sense that they all contribute some input and receive a share of the output. Two well known examples are the provision of a public good and the exploitation of a "commons," namely, a technology producing a private good under increasing marginal costs. Unrestricted access to the technology leads to severe inefficiencies due to the free rider problem in the former model, and to the tragedy of the commons in the latter.

The most interesting characterization results combine (group) strategyproofness with normative requirements of fairness, such as the Stand Alone lower bound in the provision of a public good and anonymity , i.e., equal allocations for equal messages. This property, when combined with strategyproofness, forces an "envy-free" allocation, Moulin [1993]. The results are fairly negative or at best neutral in the public good provision problem, Saijo [1991], Serizawa [1994], Ohseto [1995], but strikingly positive in the private good case, where the serial cost sharing method emerges as uniquely fair, (group) strategyproof, and not too inefficient. See Moulin and Shenker [1992], Shenker [1995], (circulated 1990). This method can also be adapted to the provision of an excludable public good—a nonrival private good. See Moulin [1994], Deb and Razzolini, [1995a], [1995b]). A general characterization of (group) strategyproof social choice functions that does not use simplifying requirements of fairness may not be too far from our reach, Shenker [1992], Moulin [1996].

6. *Concluding Comments*

The above discussion emphasizes that group strategyproofness is a tractable requirement in a number of specialized allocation problems: majority voting, possibly with phantom voters with single peaked preferences, uniform rationing of a single commodity, serial cost sharing of a private good with increasing marginal cost. Yet the negative results of the first era continue to place tight limits on the flexibility that the strategyproofness property allows to the mechanism designer. In many important contexts, such as the design of a voting rule, we must abandon the goal of strategyproofness in order to secure a minimally fair mechanism and efficient endstate. Then the mechanism designer must do her best to strike a compromise between two conflicting objectives:

- to pick a simple mechanism, with natural message spaces and easily understandable rules—the goal of exclusive procedural justice.
- to pick a mechanism implementing the desired social choice function in as many different strategic contexts as possible where the context is specified by informational and rationality

assumptions. This is the requirement of robust implementation , the sole criterion in selecting a mechanism under exclusive endstate justice.

The precise nature of this trade-off is far from clear at this point in the research effort.

Endnotes

1.Note that we could also introduce some amount of indeterminacy in the mapping f, either by adding a chance move to its domain, or by making it a multivalued correspondence.

2.This definition is not quite precise enough to yield a group strategyproof social choice function, in view of the possibility of indifferences. We omit the details.

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