

**Learning to Learn:**

The Role of Imitation and Trade in Technological Diffusion \*

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Abstract:

An endogenous growth model is developed demonstrating both static and dynamic gains from trade for developing nations due to the beneficial effects of trade on imitation and technological diffusion. The concept of learning-to-learn in both imitative and innovative processes is incorporated into a quality-ladder model with North-South trade. Domestic technological progress occurs via innovation or imitation, while growth is driven by technological advances in the quality of domestically available inputs, regardless of country of origin. In the absence of trade, Southern imitation of Northern technology leads to asymptotic conditional convergence between the two countries, demonstrating the positive effect of imitation on Southern growth. Free trade generally results in a positive feedback effect between Southern imitation and Northern innovation yielding a higher common steady-state growth rate. Immediate conditional convergence occurs. Thus, trade in this model confers dynamic as well as static benefits to the less developed South, even when specializing in imitative processes.

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## I. Introduction

The issue of whether less developed nations benefit from or are harmed by trade with industrialized nations is a recurring theme in international and development economics. Within the context of "North-South" trade, some economists focus on the effect of North-South trade when it leads to Northern specialization in industries which exhibit positive spillovers and Southern specialization in industries lacking such positive externalities (Young 1991; Stokey 1988<sup>1</sup>). Within that type of model, the less developed country (LDC) experiences negative dynamic effects which could potentially outweigh the static gains from trading with a developed country (DC). Others consider the effect of North-South trade on technological progress and diffusion (Krugman 1979; Dollar 1986; Grossman and Helpman 1991a and 1991b; Rivera-Batiz and Romer 1991;<sup>2</sup> Barro and Sala-i-Martin 1995b; Glass 1997).

This paper focuses on the effects of North-South trade on technological progress in both countries through innovative and imitative activities, paying particular attention to the modeling of imitative research. I put forth an argument that North-South trade is beneficial to the LDC, even if such trade leads to Southern specialization in imitative processes. In support of this claim, I demonstrate that in addition to static gains from trade, the LDC experiences dynamic gains from trade due to its positive effects on imitation and innovation. I do this by first showing that imitation brings about technological diffusion, thus accelerating growth in the imitating country, and second, that trade lowers the cost of imitation, thereby generally creating a positive feedback effect between Northern innovation and Southern imitation and further increased growth in both countries. Hence, the issue of North-South trade is considered not only in terms of whether such trade leads to LDC specialization in imitation, but also in terms of how such trade might increase the diffusion of technology to the South.

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<sup>1</sup>Stokey's model considers the effects of specialization in a traditional sector with no learning-by-doing versus specialization in industries with learning-by-doing, but does not set this in the context of North-South trade.

<sup>2</sup>Rivera-Batiz and Romer (1991) consider the effect of increased economic integration through trade between two similar, developed countries rather than between a developed and a less developed nation.

The model developed here most closely relates to Grossman and Helpman (1991b) and Barro and Sala-i-Martin (1995b), but makes two principal contributions to the existing literature on North-South technological diffusion. The first is an explicit modeling of imitative processes. Specifically, I introduce the notion that both imitation and innovation depend positively on past *learning-to-learn* in research, whether imitative or innovative. Thus, there is a positive externality both from past imitation and past innovation, although the spillover from innovation is assumed to be greater. Hence, learning-to-learn differs from learning-by-doing in that the skills gained are applicable to different types of research, as opposed to being limited to the exact task in which the learning occurs. Furthermore, imports of Northern capital goods and the quality of Southern infrastructure are explicitly considered in the cost of imitation for the South. If one is concerned with what is actually happening in developing countries, the modeling of imitation in terms of the effects of trade and learning-to-learn in research is particularly important. Secondly, this paper is the first to model transitional dynamics within a quality ladder model. Thus, steady-state equilibrium conditions, as well as the transitional dynamics experienced by the South as its firms successfully imitate Northern technology, are examined. Further, the possibility of the South using imitation as a springboard to leapfrog the North is considered.

The paper is organized as follows. Section II provides motivation for the modeling of imitation. To separate the effect of imitation alone from the effects of imitation with trade, the paper first focuses on imitation's contribution to technological diffusion independently of trade, and then considers the combined effects of trade with imitation. Section III therefore considers a benchmark autarky case with sufficient North-South interaction to make Southern imitation of Northern products possible at a significant cost. This case demonstrates the positive effects of imitation on growth through technological diffusion and spillovers in research. Section IV then introduces international trade, when imitation is already occurring in the South. This is compared with the previous autarky case to determine the additional effects of trade on technological progress in both countries. Section V concludes.

## **II. Imitation**

### **Learning-to-Learn**

Successful imitation allows for the diffusion of technology embodied in a product as imitators reverse-engineer that good. Moreover, imitation, like innovation, has learning-to-learn properties. In particular, successful imitation by a firm increases that firm's insight into how goods are engineered and improved upon. Thus, the higher the technology level of a good which the firm has cloned, the more likely the firm will successfully innovate the next quality level on its own. Repeated imitation therefore increases a firm's chances at successful innovation within the same product line. As an example, consider graduate studies. In many ways, our first years in graduate school are spent "reverse-engineering" the pre-existing stock of academic knowledge. During that time, we attain the skills and detailed understanding of the subject matter necessary for us to "innovate" on our own. Realistically, the learning-to-learn effects from imitation are less than those from innovation, but nevertheless can be useful as a springboard for future innovation.

### **The Effects of Trade on Imitation**

I assume that importing advanced Northern capital goods lowers the Southern cost of imitation because of general exposure to imports,<sup>3</sup> and servicing and distribution by local importing firms. For example, Lesley (1924) provides anecdotal evidence of three independent U.S. imitations of previously imported European Portland cement during the late 1800s by an individual who used Portland cement in construction, by a company which made concrete products, and by a large importing firm which distributed Portland cement within the U.S.<sup>4</sup> The importance of exposure to a good is also demonstrated in Thompson's (1992) study of 3,500 U.S. sewing machine patents, in which he finds that patenting activity followed sewing machine sales both geographically and temporally.

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<sup>3</sup> For example, if we think of each individual who is exposed to a good as having a certain probability of imitating it, then the number of people exposed to the good, and hence the volume of imports, should positively affect the overall probability that the good is imitated.

<sup>4</sup> I thank David Prentice for providing me with this example.

Further, trade provides access to international markets for successful imitators. Sokoloff (1988) finds that U.S. counties from 1790 to 1846 with access to navigable waterways had higher patenting rates than counties without such access. Moreover, he finds an increase in county patenting rates after the introduction of new waterways in or adjacent to these counties. Thus, both exposure to goods and access to larger markets appear to play important roles in research activity.

Several empirical studies consider the possible link between trade in physical goods and technological diffusion in more recent times (Eaton and Kortum 1995; Coe and Helpman 1995; Coe, Helpman, and Hoffmaister 1995). The findings of these papers support the notion that trade contributes significantly to technological diffusion, although the exact mechanism is not specified. However, using the Coe and Helpman (1995) data, Keller (1997) finds evidence of international R&D spillovers using randomly generated bilateral trade shares, casting doubt on the importance of trade in goods as the channel for technological diffusion when considering similar, interrelated countries. Still, Ben-David (1996) finds that trade-based country groupings are more likely to converge than randomly selected country groupings. Ben-David and Rahman (1996) build upon this result by suggesting that convergence in trade-based country groupings can be attributed to convergence in technologies. This is demonstrated by a high incidence of total factor productivity convergence among trade-based country groups, whereas no such technological convergence is found among randomly selected country groupings. Thus trade appears to play an important role in technological diffusion and, in turn, conditional convergence.

Figure 1 plots imports of capital goods from developed countries against a proxy of imitative activity for a pooled cross section of 51 countries in 1970-74, 1975-79, and 1980-84. Since these data (described in the Appendix) are at a very aggregate level, they cannot say much about the micro level interactions between trade and imitation considered here. However, the strong correlation shown in this graph is suggestive of interaction between imports of high technology goods and imitation. Empirical work completed in Connolly (1997) finds a significant positive relationship between imports of high technology goods as a share of GDP and proxies for domestic innovation and imitation.



the accumulation of human capital through learning-by-doing in research. Here however, I focus solely on the role of technological diffusion.<sup>6</sup>

Following conventional notation for rising product quality models (Grossman and Helpman 1991a; Aghion and Howitt 1992; Barro and Sala-i-Martin 1995a), there are a fixed number,  $J$ , of intermediate goods, whose quality levels are improved upon through innovation (or imitation).  $q$  denotes the size of quality improvements with each innovation of a particular good, where  $q$  is assumed to be an exogenously determined constant greater than 1. The particular rung of the quality ladder at which a good of type  $j$  is located is indicated by  $k_j$ . Hence, normalizing so that all goods begin at quality 1, each subsequent innovation will make the good  $q$  times more productive than its predecessor. Thus, the quality levels of a good in sector  $j$  will rise from 1 to  $q$  with the first innovation, to  $q^2$  with the second innovation, and to  $q^{k_j}$  with the  $k_j$ th innovation.

Using the basic setup of Barro and Sala-i-Martin (1995a), consider the following aggregate final goods production function in the North, a DC (country 1), and in the South, an LDC (country 2):

$$(1.1) \quad Y_1 = A_1 L_1^a \sum_{j=1}^J (q^{k_{1j}} x_{1k_j})^{1-a}$$

$$Y_2 = A_2 L_2^a \sum_{j=1}^J (q^{k_{2j}} x_{2k_j})^{1-a}, \quad \text{where } 0 < a < 1.$$

The final goods industry is comprised of many perfectly competitive firms.  $A_i$  is a productivity parameter dependent upon country  $i$ 's institutions. Since the North is the more developed country, its institutions such as tax laws, property rights, and government services which affect productivity are, by definition, better than their Southern counterparts so  $A_1 > A_2$  by assumption.  $L_i$  is the labor input used by the representative firm for final goods production and  $(q^{k_{ij}} x_{ik_j})$  is the quality adjusted amount of intermediate goods of type  $j$ , used in country  $i$ . Hence, as the quality level of intermediate goods rises,

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experience technological progress at a rate less than or equal to its autarky rate. Hence, the LDC will face dynamic losses from trade which could possibly outweigh the static gains from trade with a DC.

<sup>6</sup>For seminal treatments of the issue of human capital accumulation and technological diffusion, see Nelson and Phelps (1966) and van Elkan (1996).

so does final goods output,  $Y_i$ , which is assumed to be different in each country. Let the final good be numeraire in each country.<sup>7</sup>

In this section there is no international trade and no *costless* international diffusion of technology. Each country is dependent upon its own technology level for intermediate goods production. As is consistent with the empirical findings of Mansfield, Swartz, and Wagner (1981), I assume that the cost of imitation is less than the cost of innovation. Since Northern technology is more advanced than Southern technology, Northern firms initially innovate and Southern firms initially imitate Northern technology, at least until the gap in their technology levels is eliminated.

Once knowledge of how to produce an intermediate good exists domestically, it can be produced using the final goods production function. Therefore, the marginal cost of producing an intermediate good equals the marginal cost of producing a final good, which, due to perfect competition in the final goods industry, equals the price of the final good (i.e.  $MC_i = P_{Y_i} = 1$ ). Thus, the marginal cost of producing an intermediate good is independent of its quality level and is identical across all domestic sectors.

I also assume that knowledge of how to make a good is public knowledge within a country. One could think of countries as having domestically enforced patents which protect the lead firm's domestic monopoly of that quality good, while at the same time costlessly disseminating acquired knowledge to other domestic firms. Since the last innovator in each sector is the only firm legally allowed to produce the highest quality intermediate good, it will use limit pricing to wipe out sales of lower quality intermediate goods in its sector.<sup>8</sup> In either country  $i$ , lead firms in all intermediate goods sectors choose a price  $\epsilon$  less than the limit price,  $P_i = qMC_i$ , where  $MC_i = 1$  and  $\epsilon$  is arbitrarily small.<sup>9</sup> At this limit price, implied demand (derived by maximizing profits for final goods production) for intermediate goods in sector  $j$  is

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<sup>7</sup>When trade is considered in Section IV, the Northern final good will be used as numeraire.

<sup>8</sup>Depending on whether  $q(1-\alpha)$  is greater than or less than the marginal cost of production, the lead firm will respectively use monopoly or limit pricing to capture the entire domestic market for its sector. I assume  $q(1-\alpha) < MC_i$ , implying that limit pricing will be used by all lead firms.

<sup>9</sup>This limit price reflects the fact that the lead intermediate good in any sector is  $q$  times more efficient than the second best domestically available good. Since the lowest price the producer of the second best

$$x_{ik_j} = L_i [A_i (1-a) q^{k_{ij}(1-a)} \frac{P_{Y_i}}{P_i}]^{1/a} = L_i A_i^{1/a} \left(\frac{1-a}{q}\right)^{1/a} q^{k_{ij}(1-a)/a}.$$

Let  $Q_i = \sum_{j=1}^J q^{k_{ij}(1-a)/a}$  represent an aggregate domestic quality index. Then aggregate demand for intermediate goods across all sectors can be expressed in terms of  $Q_i$ :

$$(1.2) \quad X_i = L_i A_i^{1/a} \left(\frac{1-a}{q}\right)^{1/a} Q_i.$$

Substituting the demand for intermediate goods into the aggregate production function in equation (1.1) yields aggregate output per worker:

$$(1.3) \quad \frac{Y_i}{L_i} = A_i^{1/a} \left(\frac{1-a}{q}\right)^{(1-a)/a} Q_i, \quad \text{where } 0 < a < 1.$$

Taking  $A_i$  and  $L_i$  as given, each country's output depends solely on  $Q_i$ . Hence, differences in the countries' growth rates depend solely on their respective rates of technological progress, either through innovation or through imitation. We therefore turn our attention to deriving the equilibrium rates of Northern innovation and Southern imitation.

### **Resources Devoted to Northern Innovation Under Autarky.**

At each point in time within an intermediate goods industry, there is a lead firm which produces the highest quality intermediate good and follower firms which have temporarily exited the market. Combined with the assumption of costless domestic technological diffusion, this implies that under autarky only followers will undertake research and continual leapfrogging will occur within each sector.<sup>10</sup>

When deciding what resources to devote to research, a potentially innovating firm considers the expected present value of profits its innovation would earn. Under autarky,

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good can charge is  $MC_i$ , the lead firm can successfully capture the entire market for this type of good by selling at any price slightly below  $qMC_i$ .

<sup>10</sup>As shown in the Appendix, this occurs since both leaders and followers face the same costs and probabilities of bringing about the next innovation, while lead firms have less incentive to bring about the next innovation since doing so will eliminate or at least reduce their profits from earlier innovations.

Northern innovators are unaffected by potential Southern imitation. However, the Northern leader's flow of profits are eliminated when the next Northern innovation occurs, causing the firm to lose its leadership position. Hence, the present value of the flow of profits depends on the probability that the next innovation occurs.

Within a Northern intermediate goods sector  $j$ , presently at quality level  $k_{Ij}$ ,  $p_{IIk_j}$  is the probability per unit of time that the next  $(k_{Ij}+I)$ th innovation occurs. Specifically,  $p_{IIk_j}$  follows a Poisson process which depends positively on resources devoted to research,  $z_{Ik_j}$ , and past domestic learning-to-learn,  $J_{Ik_j}$ , in industry  $j$ , and negatively on the complexity,  $f(k_{Ij})$ , of the good upon which firms are attempting to improve:

$$(1.4) \quad p_{IIk_j} = z_{Ik_j} f(k_{Ij}) J_{Ik_j}, \quad \text{where}$$

$$J_{Ik_j} = \max ( \mathbf{b}_C q_C^{k_{Ij}}, \mathbf{b}_I q_I^{k_{Ij}} ), \quad \mathbf{b}_I > \mathbf{b}_C > 0, \quad \text{and}$$

$$f(k_{Ij}) = (1/z_{II}) q^{-k_{Ij}/a}.$$

Subscripts  $C$  and  $I$  denote copying and innovation, respectively.  $J_{Ik_j}$  reflects the positive spillover effects of past learning-to-learn through imitation or innovation. For a particular domestic intermediate goods sector  $j$ ,  $q_C^{k_{Ij}}$  is the highest quality level attained through imitation and  $q_I^{k_{Ij}}$  is the highest quality level attained through innovation. If the country has no imitative experience, then  $q_C^{k_{Ij}} = 0$ , and if the country has no innovative experience, then  $q_I^{k_{Ij}} = 0$ .<sup>11</sup>  $\mathbf{b}_C$  and  $\mathbf{b}_I$  are positive coefficients on past imitative and innovative experience, respectively.  $\mathbf{b}_I > \mathbf{b}_C$  since there should be greater learning-to-learn effects in innovation than in imitation. Finally,  $f(k_{Ij})$  reflects the increased difficulty of innovation when higher quality levels are being invented, implying a lower probability of success, all else equal. This term further includes a country specific fixed cost of innovation,  $z_{II}$ .

The form assumed for the probability of innovation guarantees constant returns to innovative research with respect to current technology levels ( $k_{Ij}$ ). This assumption is necessary to consider a balanced growth path for the North. Furthermore this assumption

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<sup>11</sup> I assume that each country has experience in at least one type of research. If not, then  $q = I$ .

is reasonable if we believe that there are an infinite number of potential innovations and hence, there need not be diminishing returns to innovative R&D (Romer 1990).

Using the expected flow of net profits from innovative research, combined with the assumption of free entry into research yields (as derived in the Appendix) an expression for the probability of innovation which holds for all Northern intermediate goods sectors and is independent of  $k_{Ij}$ :

$$(1.5) \quad p_{II} = \frac{\mathbf{b}_I}{\mathbf{z}_{II}} \mathbf{Y}_I - r_I,$$

where  $\mathbf{Y}_I = L_I A_I^{1/a} (1-\mathbf{a})^{1/a} MC_I (q-1)/q$ . This is the equilibrium value for  $p_{II}$  once the equilibrium interest rate,  $r_I^*$ , is entered into this expression.  $r_I^*$  will later be shown to be constant, thus insuring that the equilibrium probability of innovation is also constant.

Equilibrium aggregate resources devoted to Northern innovation are found by substituting equation (1.4) into (1.5) and aggregating across all intermediate good sectors:

$$(1.6) \quad \mathbf{Z}_{II} = Q_I \left( \mathbf{Y}_I - r_I \frac{\mathbf{z}_{II}}{\mathbf{b}_I} \right).$$

Hence, aggregate resources devoted to innovation by Northern follower firms depend positively on domestic market profitability and the aggregate quality index,  $Q_I$ , and negatively on the interest rate and the experience-adjusted cost of innovative research,  $\mathbf{z}_{II}/\mathbf{b}_I$ . Thus, aggregate resources devoted to Northern research are a constant multiple of the Northern technology level, as is the aggregate market value of Northern firms (shown in the Appendix). Note however, that the increased resources devoted to research as the aggregate quality index rises, do not lead to greater rates of innovation, but rather are needed to offset the greater difficulty of innovating as quality levels increase.

### **Resources Devoted to Southern Imitation under Autarky**

All Southern firms involved in R&D undertake imitative research so long as the cost of imitation is less than or equal to that of innovation. Research is again undertaken by followers, and continual leapfrogging occurs within each intermediate goods sector.

Hence, each potential imitator considers the expected present value of profits successful imitation would yield, which depends on the probability of the next successful imitation, which would wipe out the firm's profits.

Similarly to the probability of innovation, the probability per unit of time,  $p_{C2k_j}$ , that an intermediate good of quality rung  $(k_j+1)$  is copied in the South depends positively on resources spent by Southern firms in terms of output devoted to reverse engineering,  $z_{C2k_j}$ , negatively on the complexity,  $f_C(k_j)$ , of the good which is being copied, and positively on past learning-to-learn in that domestic industry:

$$(1.7) \quad p_{C2k_j} = z_{C2k_j} f_C(k_j) J_{2k_j}, \quad f'(k_j) < 0, \text{ where}$$

$$J_{2k_j} = \max(\mathbf{b}_C q_C^{k_{2j}}, \mathbf{b}_I q_I^{k_{2j}}), \quad \mathbf{b}_I > \mathbf{b}_C > 0,$$

$$f_C(k_{2j}) = \frac{1}{z_{C2} \hat{q}_j^s (e^{-w} + 1)} q^{-k_{2j}/a}, \quad \mathbf{s} > 0, \quad \text{and} \quad \hat{q}_j = \frac{q^{k_{2j}}}{q^{k_{1j}}}.$$

Relative to the cost of innovation, two new factors enter into the cost of imitation,  $z_{C2} \hat{q}_j^s (e^{-w} + 1)$ . The first term,  $z_{Ci}$ , parallels the fixed cost in innovative research,  $z_{Ii}$ . The second term,  $\hat{q}_j^s$ , depends on the South/North technology ratio in sector  $j$  and reflects the increasing cost of imitation as Southern technology approaches that of the North. Hence, there are decreasing returns to imitation as the South imitates more and more of the existing Northern inventions, thereby decreasing the pool of potential goods to imitate. Finally, the third term,  $(e^{-w} + 1)$ , reflects lower costs of gathering information about foreign goods with greater interaction,  $w$ , between the two countries, as measured by imports of capital goods,  $M$ , and the quality of communications and transportation infrastructure,  $F$ :  $w = I_1 M + I_2 F$ .<sup>12</sup>

Since the cost of imitation is increasing as the North-South technology gap decreases, there will be a transition path for the South before reaching steady-state. To analyze this transition path, I consider a representative industry (denoted by the subscript  $a$ ) which is defined as an average of all intermediate goods industries. This is needed to avoid any jumpiness in quality improvements that would occur if contemplating the

behavior of an individual sector. The cost of imitation faced by the average intermediate goods industry therefore depends on the aggregate South/North technology ratio,  $\hat{Q}$ :

$$z_{c2_a}(\hat{Q})^s (e^{-w} + 1), \quad \text{where } s > 0 \quad \text{and} \quad \hat{Q} = \frac{Q_2}{Q_1}.^{13}$$

Let us first consider steady-state conditions for both countries, and then derive the transitional dynamics for the South.

### Steady-State under Autarky

Since the probability of innovation in a single Northern intermediate good sector is  $p_{11}$ , and the proportionate change in  $Q_1$  due to a single innovation is  $q^{(1-a)/a} - 1$ , the expected proportionate change in  $Q_1$  per unit of time is

$$(1.8) \quad E\left(\frac{DQ_1}{Q_1}\right) = p_{11}(q^{(1-a)/a} - 1).$$

Each intermediate goods sector experiences innovations at random time intervals, leading to uneven increases in the quality level within each sector. However, assuming a sufficiently large number of independent intermediate goods sectors so that the Law of Large Numbers holds, the aggregate quality index will grow in a smooth manner. Then equation (1.8) can be used to approximate  $\dot{Q}_1/Q_1$ . Substituting equation (1.5) for  $p_{11}$  into equation (1.8) yields

$$(1.9) \quad \frac{\dot{Q}_1}{Q_1} = (q^{(1-a)/a} - 1) \left[ \frac{b_1}{z_{11}} Y_1 - r_1 \right].$$

The above term will drive Northern output growth.

Similarly, an expression for Southern technological progress can be found using the steady-state probability of imitation for the representative Southern industry. In steady-state, the North-South technological gap is constant, implying that the number of

<sup>12</sup>The levels of  $M$  and  $F$  are taken as given, with  $M = 0$  in this autarky case.

<sup>13</sup>Of course, if  $\hat{Q}$  becomes greater than 1, the representative Southern industry will be innovating.

goods which can potentially be imitated is also constant. Thus, while imitative research exhibits decreasing returns to scale during transition to steady-state, it faces constant returns to scale in steady-state.<sup>14</sup> Since  $J_{C2k_a} = \mathbf{b}_c q^{k2a}$ , and assuming free entry into research, the probability of successful imitation for the representative Southern sector is

$$(1.10) \quad p_{C2k_a} = \frac{\mathbf{b}_c}{\mathbf{z}_{c2} (\hat{Q}^*)^s (e^{-\mathbf{w}} + 1)} \mathbf{Y}_2 - r_2 .$$

Notice that when  $\mathbf{w}$  is high (i.e., when there is a good communications and transportation infrastructure level), the cost of imitation is lower, leading to a higher probability of imitation. Similarly, the smaller the steady-state North-South technology gap, the lower the probability of successful imitation, all else equal. Given this steady-state probability of imitation, growth in aggregate Southern technology is

$$(1.11) \quad \frac{\dot{Q}_2}{Q_2} = (q^{(1-a)/a} - 1) \left[ \frac{\mathbf{b}_c}{\mathbf{z}_{c2} (\hat{Q}^*)^s (e^{-\mathbf{w}} + 1)} \mathbf{Y}_2 - r_2 \right] .$$

### Household Optimization

In each country, infinitely lived households maximize a constant intertemporal elasticity of substitution utility function:

$$\int_0^{\infty} e^{-\mathbf{r}t} (c_{it}^{1-\mathbf{q}} - 1)/(1 - \mathbf{q}) dt,$$

where  $c_{it}$  is consumption per capita at time  $t$ , and  $\mathbf{r}$  and  $(1/\mathbf{q})$  are respectively, the subjective discount rate and the elasticity of the intertemporal substitution common to both countries. This optimization yields the usual expression for consumption growth:

$$(1.12) \quad \frac{\dot{c}_i}{c_i} = \frac{1}{\mathbf{q}} (r_i - \mathbf{r}) .$$

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<sup>14</sup>This parallels Evenson and Kislev's (1976) model with two distinct types of research activity. The first is basic research which advances basic knowledge and determines the distribution within which the second type of research, applied research, is able to search. The authors argue that if basic research stops, there will be diminishing returns to applied research. However, if both basic and applied research persist at constant rates, then in steady-state, there will be constant returns to applied research.

The aggregate budget constraint for each country is given by  $Y_i = C_i + X_i + Z_i$ , where  $C_i$ ,  $X_i$ , and  $Z_i$  are the total resources devoted to consumption, production of intermediate goods, and research, respectively. Solving for  $C_i$  and substituting equations (1.3) and (1.2) for  $Y_i$  and  $X_i$ , and (1.5) and (1.6) for  $Z_i$ , we see that  $C_i$  is a constant multiple of  $Q_i$ :

$$C_i = Q_i \left\{ L_i \left[ 1 - \left( \frac{1-a}{q} \right) \right] - p_{ii} \frac{z_{ii}}{b_i} \right\},$$

where  $L_i = L_i A_i^{1/a} ((1-a)/q)^{(1-a)/a}$ . Similarly,  $C_2$  is a constant multiple of  $Q_2$ . Thus, in equilibrium, both countries' aggregate consumption, output, intermediate goods demand, and resources devoted to research are all constant multiples of the domestic technology level,  $Q_i$ . So the growth rates of each of these variables is equal to the domestic growth rate of  $Q_i$ . Let us refer to this growth rate as  $g_i$ . Technology growth for both countries is derived from firms' market behavior, and consumption growth is derived from household optimization. Thus, we set the growth rate of consumption equal to that of domestic technology to find the steady-state interest rate in each country:

$$(1.13) \quad r_i^* = r + q \left( \frac{b_i}{z_{ii}} Y_i - r \right) / \left( \frac{1}{q^{(1-a)/a} - 1} + q \right)$$

$$r_2^* = r + q \left( \frac{b_c}{z_{c2} (\hat{Q}^*)^s (e^{-w} + 1)} Y_2 - r \right) / \left( \frac{1}{q^{(1-a)/a} - 1} + q \right).$$

To return to growth rates, subtract  $r$  from the above equations and divide by  $q$ :

$$(1.14) \quad g_i^* = \left( \frac{b_i}{z_{ii}} Y_i - r \right) / \left( \frac{1}{q^{(1-a)/a} - 1} + q \right),$$

$$g_2^* = \left( \frac{b_c}{z_{c2} (\hat{Q}^*)^s (e^{-w} + 1)} Y_2 - r \right) / \left( \frac{1}{q^{(1-a)/a} - 1} + q \right),$$

where  $Y_i = L_i A_i^{1/a} (1-a)^{1/a} MC_i (q-1)/q$ , and  $MC_i = 1$ .

Hence, Northern growth depends positively on the spillover from past innovative experience and domestic market profitability, and negatively on the fixed cost of innovation. Similarly, the steady-state growth rate of the South depends positively on the spillover from past imitative experience and negatively on the cost of imitation.

For Southern growth to remain constant in steady-state,  $\hat{Q}$  must be constant, i.e. Southern imitation must occur at the same rate as Northern innovation. Both countries therefore grow at the same rate in steady-state. So even though there is no international trade or capital flows, there *is* interest rate equalization between the two countries in steady-state. Thus, international technological diffusion is sufficient to bring about interest rate equalization in steady-state.

Setting steady-state growth rates in both countries equal to each other yields an expression for the steady-state South/North aggregate quality ratio

$$\hat{Q}^* = \left\{ \frac{Y_2}{Y_1} \frac{z_{11}}{z_{c2}(e^{-w} + 1)} \frac{b_c}{b_1} \right\}^{1/s} = \left[ \frac{L_2}{L_1} \left( \frac{A_2}{A_1} \right)^{1/a} \frac{MC_2}{MC_1} \frac{z_{11}}{z_{c2}(e^{-w} + 1)} \frac{b_c}{b_1} \right]^{1/s}.$$

This expression reflects the relative profitability of R&D in both countries under autarky. Note also that a higher  $w$  (i.e., greater interaction with the North) implies a lower cost of imitation and a higher steady-state South/North technology ratio. If  $\hat{Q}^*$  is less than one (i.e.,  $Q_2^* < Q_1^*$ ), the South will remain the imitating country. While the South will in steady-state grow the North, its income level will remain below that of the North. In other words, conditional convergence results. On the other hand, if  $\hat{Q}^*$  is greater than one (i.e.,  $Q_2^* > Q_1^*$ ), then the South converges with the North in absolute terms and takes over the role of lead innovating country once  $\hat{Q}$  equals one.<sup>15</sup> This will be a one-time switch with the South remaining in the leadership position from that point on since the South must have the comparative advantage in innovation in order to leapfrog the North.<sup>16</sup> The

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<sup>15</sup> The previous growth equations would then continue to hold with reversed country subscripts.

<sup>16</sup> This one-time switch differs from leapfrogging models which consider the existence of different avenues for technological development. In those types of models, relative lack of experience helps countries or firms adopt new technologies, thereby leapfrogging more experienced competitors who had

steady-state growth rate of both countries will then be determined by the South's innovation rate, which will be greater than that of the North when innovating.

Previous assumptions with regard to institutions ( $A_1 > A_2$ ) and learning-to-learn ( $\mathbf{b}_I > \mathbf{b}_C$ ) tend to keep the North in the leadership position. On the other hand, the assumption that the Southern labor force is larger than the Northern labor force, favors the profitability of research in the South under autarky. Since I am interested in the effects of North-South trade when the South specializes in the research activity with smaller positive spillovers, I assume for the rest of the paper that the North has the long-run comparative advantage in innovation, and therefore the South remains the imitating country forever.

### **Southern Imitation versus Innovation under Autarky**

Under autarky, Southern imitation does not affect the North. However, the decision to imitate Northern technology instead of trying to innovate on its own greatly benefits the South. In particular, if the South chose to innovate rather than imitate it would, in steady-state, grow at a rate dependent upon  $(\mathbf{b}_I / \zeta_{I2})$  instead of  $(\mathbf{b}_C / \mathbf{z}_{C2} \hat{Q}^S (e^{w+1}))$ . Since the North by definition has a more efficient R&D sector (i.e.  $\zeta_{I2} > \zeta_{I1}$  and  $A_1 > A_2$ ), it grows more quickly than the South if both are independently innovating. Hence, divergence would occur between the two countries. However, if the South imitates, both countries experience the same steady-state growth rate, determined by Northern innovation. Thus, the South enjoys a higher steady-state growth rate under autarky with imitation than under autarky with innovation. Moreover, with imitation, the South grows more quickly than the North during its transition to steady-state.

### **Transitional Dynamics in the South**

Since the North is unaffected by Southern imitation under autarky, it is always in steady-state. However, Southern interest and growth rates change with the North-South technology gap, implying a transition path to steady-state for the average Southern

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specialized in now outmoded technologies (Brezis, Krugman, and Tsiddon 1993; Ohyama and Jones 1995). In these types of models, it is therefore possible for continual leapfrogging to occur.

intermediate goods industry,  $a$ . Consider the following expression for the expected present value of profits for the lead Southern imitator in the average sector  $a$ :

$$E(v_{C2k_a}) = \mathbf{p}_{C2k_a} \int_t^\infty \exp[-\int_t^s (r_2(v) + p_{C2k_a}(v))dv] ds,$$

where  $r_2(v)$  is the rate of return in the South at time  $v$  and  $p_{C2k_a}(v)$  is the probability at time  $v$  that the intermediate good of quality rung  $(k_{2_a} + 1)$  is copied. Free entry into research implies that at all times, the expected present value of profits from successful imitation times the probability of success at imitation must equal the resource cost of imitation:

$$p_{C2k_a} E(v_{C2,k_a+1}) = p_{C2k_a} \{ \mathbf{p}_{C2,k_a+1} \int_t^\infty \exp[-\int_t^s (r_2(v) + p_{C2k_a+1}(v))dv] ds \} = z_{C2k_a}.$$

Differentiating both sides of the above equation (using Leibniz's rule for the left-hand side) yields an expression for the Southern interest rate

$$(1.15) \quad r_2 = \frac{\dot{z}_{C2k_a}}{z_{C2k_a}} - \frac{\dot{p}_{C2k_a}}{p_{C2k_a}} - \frac{\dot{\mathbf{p}}_{C2,k_a+1}}{\mathbf{p}_{C2,k_a+1}} + (p_{C2k_a} \mathbf{p}_{C2,k_a+1} / z_{C2k_a}) - p_{C2,k_a+1}$$

$$= \mathbf{s} \frac{\dot{\hat{Q}}}{\hat{Q}} + (p_{C2k_a} \mathbf{p}_{C2,k_a+1} / z_{C2k_a}) - p_{C2,k_a+1},$$

where the first term is a capital gains term, the second is a dividend term, and the third reflects the Schumpeterian concept of creative destruction caused by the next imitation.<sup>17</sup>

The transition path for the South, derived in the Appendix, is described by a system of autonomous differential equations in the variables,  $\hat{Q}$  and  $\mathbf{c}_2$ , where  $\mathbf{c}_2 = C_2/Q_2$ :

$$\frac{\dot{\hat{Q}}}{\hat{Q}} = \frac{\dot{Q}_2}{Q_2} - \frac{\dot{Q}_1}{Q_1} = \frac{\mathbf{b}_c}{z_{C2} \hat{Q}^s (e^{-w} + 1)} (q^{(1-a)/a} - 1) \{ \mathbf{L}_2 [1 - (\frac{1-a}{q})] - \mathbf{c}_2 \} - \gamma_1, \text{ and}$$

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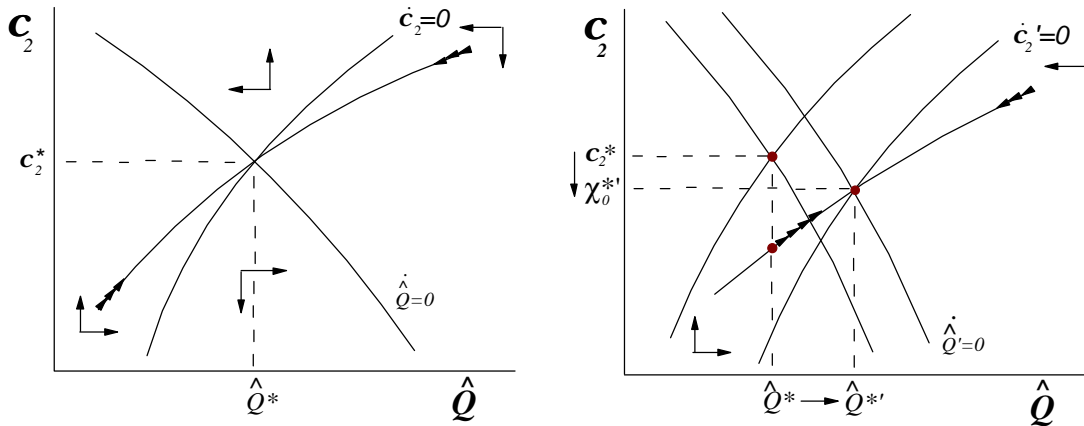
<sup>17</sup> This holds since  $\frac{\dot{z}_{C2k_a}}{z_{C2k_a}} = \frac{\dot{\cdot}}{k_a} \frac{1-a}{a} \ln q + \frac{\dot{p}_{C2k_a}}{p_{C2k_a}} + \sigma \frac{\dot{\hat{Q}}}{\hat{Q}}$ , and  $\frac{\dot{\mathbf{p}}_{C2,k_a+1}}{\mathbf{p}_{C2,k_a+1}} = \frac{\dot{\cdot}}{k_a} \frac{1-a}{a} \ln q$ .

$$\frac{\dot{c}_2}{c_2} = \frac{\dot{C}_2}{C_2} - \frac{\dot{Q}_2}{Q_2} = \frac{b_c}{z_{c_2} \hat{Q}^s (e^{-w} + 1)} \left\{ \left( \frac{s}{q} - 1 \right) \left( q^{(1-a)/a} - 1 \right) \left( L_2 \left( 1 - \frac{1-a}{q} \right) - c_2 \right) + \right.$$

$$\left. \frac{L_2}{q} q^{(1-a)/a} \left( \frac{1-a}{q} \right) (q-1) MC_2 \right\} - \frac{1}{q} (s \gamma_1 + p_{C_2, k_{a+1}} + r).$$

$\dot{Q}=0$  is downward sloping in  $(\hat{Q}, c_2)$  space. Furthermore, as  $\hat{Q}$  increases (causing a rise in the cost of imitation),  $\dot{Q}$  decreases. The slope of  $\dot{c}_2=0$  depends on  $s$  and  $q$ . If  $s > q$ ,  $\dot{c}_2=0$  is downward sloping and an increase in  $c_2$  leads to a decrease in  $\dot{c}_2$ . On the other hand, if  $s < q$ ,  $\dot{c}_2=0$  is upward sloping and an increase in  $c_2$  leads to an increase in  $\dot{c}_2$ . In either case, there is an upward sloping stable saddle path.<sup>18</sup> For brevity, only the case of

**Figure 2. ( $s < q$ )**



$s < q$  is graphed. If  $\hat{Q}$  is initially below its steady-state value,  $\hat{Q}$  and  $c_2$  rise monotonically until they reach their steady-state values. During this transition,  $\dot{Q}/\hat{Q}$  is positive but decreasing monotonically toward zero. This implies that the interest rate is initially higher and technology is rising faster in the South than the North during the transition. However, as the pool of intermediate goods left to be copied dwindles, diminishing returns to imitation set in. This lowers growth in  $\hat{Q}$  and reduces the Southern interest rate until it equals the Northern interest rate in steady-state. So the South grows more quickly than the North during the transition but slows down until steady-state is reached. Thus,

<sup>18</sup> This holds when both loci are downward sloping since the  $\dot{c}_2=0$  locus is steeper than the  $\dot{Q}=0$  locus.

asymptotic conditional convergence results, as is consistent with empirical findings of Barro and Sala-i-Martin (1991). Similarly to  $\dot{\hat{Q}}/\hat{Q}$ ,  $\dot{c}_2/c_2$  is also positive but decreasing during the transition. This implies that consumption is initially growing more quickly than technology in the South. However, consumption growth falls more quickly than  $Q_2$  growth until they are equal in steady-state.

As an experiment, consider what happens if  $w$  increases due to an exogenous increase in the quality of Southern infrastructure. This would cause the  $\dot{\hat{Q}} = 0$  locus to shift up and right. When  $s > q$ , the  $\dot{c}_2 = 0$  locus shifts up and right, and when  $s < q$ , it shifts down and right. In either case,  $C_2$  immediately jumps down to the new stable saddle path, and then rises gradually until it reaches its new steady-state value. Since the aggregate quality level does not instantaneously jump up, this initial fall in the consumption/quality ratio reflects an initial fall in consumption as investors shift expenditures away from consumption toward imitative research with the rise in the Southern interest rate. Even though  $C_2$  is increasing during the transition, it will approach a new lower steady-state value. Still, the initial fall in consumption is temporary. Steady-state consumption is greater than before the increase in  $w$ . Interestingly, this points to a conflict between the short and long run consequences of an improvement in infrastructure for Southern consumption. In terms of technology,  $\hat{Q}$  rises to a new higher steady-state level. Thus, increased North-South interaction leads to a smaller steady-state technology gap between the two countries, and could lead to the South taking over as the lead innovating country if this pushes  $\hat{Q}^*$  above 1.

It is tempting to say that trade liberalization, which would also cause an increase in  $w$ , would have similar effects. However, this would ignore the interaction of imitation and innovation once trade is allowed. For this reason, I now open up the two countries to free trade to properly analyze the effects of trade on both imitation and innovation.

#### **IV. Imitation with Trade**

Free trade is introduced in this section, assuming that the two countries are starting from their previous steady-state positions with the North innovating and the South

imitating. Let the Northern final good be numeraire. Since I am interested in the case where the South specializes in imitative activities under free trade with the North, I assume that the marginal cost of production is greater in the North than in the South ( $I = MC_1 > MC_2$ ).<sup>19</sup> I further assume that trade is balanced so there are no international capital flows. With free trade, firms can now use imports of intermediate goods in final goods production. Hence, Southern firms will import any intermediate goods that had not yet been copied, and export the Southern final good, as well as any lead intermediate goods which they had previously copied.<sup>20</sup> Since importing firms are responsible for distributing and servicing these intermediate goods, they learn a great deal about the goods they are selling. Specifically, they learn which products are in greatest demand, what are the most recent developments within the industry, how to adapt the goods to local conditions if necessary, and how to fix or replace the goods they sell. Hence, importing intermediate goods lowers the cost of imitation. Thus, for a given infrastructure level and past learning-to-learn, countries with higher import levels face lower costs of imitation.

When the two countries open up to free trade, their technological gap may be positive or zero. Northern firms are now concerned with the joint probability of losing their market either to the next innovation or to a lower-priced imitation. The expected profits for Northern intermediate goods firms therefore now also depend on the probability of successful imitation. Furthermore, since the South can immediately import higher quality Northern intermediate goods for use in final goods production, it is no longer limited by its own ability to produce intermediate goods. This implies that Southern growth will be determined by Northern technological progress.

Once faced with Northern competition, Southern intermediate goods firms must decide whether to simply import intermediate goods or initially import but then try to imitate lead Northern goods. Imitating firms again consider the expected present value of profits to determine the amount of resources to devote to imitative research. Relative to autarky when Southern firms had only to consider imitating goods one quality rung above the current *Southern* quality level, Southern firms must now contemplate imitating

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<sup>19</sup>This is analogous to assuming a larger labor force in the South than in the North.

Northern goods possibly several quality rungs above their own experience level. Consequently, the total resources required to bring about successful imitation of lead goods that are several quality levels ahead of the Southern firm's experience are greater than in autarky where Southern firms were moving up the quality level one rung at a time. These higher resource costs reflect the assumption that a Southern firm must imitate all quality levels between its current domestic level and the lead Northern good it is targeting. Hence, there is a tradeoff involving the North-South technology gap. A larger gap implies a lower cost of imitation for each level being imitated, but it also implies that many more levels must be imitated before profits can be earned by the Southern firm under free trade.

Thus, Southern firms in sectors with a relatively small initial North-South technology gap will undertake imitative research since the expected present value of profits will be greater than or equal to the expected research costs. However, firms in sectors that are far behind their Northern counterparts will not devote any resources to imitation since for them the resources needed for imitation would be greater than the expected present value of profits. I consider two extreme cases, that of an initial value of  $\hat{Q}$  which is so low that *no* Southern firms will imitate, and that of an initial value of  $\hat{Q}$  equal to one so that *all* Southern firms decide to imitate once free trade begins.

### **No Southern Imitation**

If  $\hat{Q}$  is very low when international trade begins, then Southern firms know that the expected profits from successful imitation of lead goods are less than their expected research expenditures. Hence, they will simply import the goods from the North. Since the South can immediately import and use higher quality Northern intermediate goods, there is a level jump in Southern output. This can be thought of as the static gain from trade. Furthermore, in that situation, Northern innovating firms need not worry about potential imitation of their goods. Northern firms therefore behave exactly as in the previous autarky case, except that greater expected profits (due to increased market size

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<sup>20</sup>Since the Southern firm can underprice the Northern lead firm, both countries will switch to using copied intermediate goods in their final goods production whenever they are available.

under free trade) lead to a greater rate of innovation. Thus, the growth rate in both countries is unambiguously greater than under autarky:

$$g_l = g_2 = \left[ \frac{b_l}{z_{ll}} (L_1 A_1^{1/a} + L_2 A_2^{1/a}) (1-a)^{1/a} MC_l \frac{q-1}{q} - r \right] / \left( \frac{1}{q^{(1-a)/a}} + q \right), \quad MC_l = 1.$$

Northern technology will grow at the above rate. However, since Southern firms are no longer imitating, Southern technology will stop growing altogether. As a result, technological divergence occurs between the two countries. However this is not a bad outcome for the South since trade allows it to immediately enjoy the benefits of higher quality Northern intermediate goods and insures immediate conditional convergence with the North at a new higher steady-state growth rate. Thus, free trade is a clear improvement for both countries relative to autarky.

### **Imitation in All Southern Intermediate Goods Sectors with Free Trade**

Now suppose that *all* Southern intermediate good sectors had imitated lead Northern goods prior to opening up to free trade, but the North remains the lead innovating country due to a comparative advantage in the cost of innovation. Then, when trade begins, lead imitating Southern firms immediately capture the world market for intermediate goods due their marginal cost of production advantage. As soon as the next round of Northern innovation displaces the lead Southern imitation in a given sector, Southern firms will begin attempting to imitate this new quality level. This Vernon-type product cycle will continue in all sectors. Since the Southern cost of imitation is less than the Northern cost of innovation, Southern firms will keep pace with Northern innovations, often having to wait for the next innovation to occur before imitating again with a lag. Furthermore, with many intermediate good sectors, the Law of Large Numbers implies that the aggregate South/North quality level ratio will remain approximately constant and equal to 1 in this situation.<sup>21</sup> Hence, there is no transition path. Steady-state conditions are derived as before using equilibrium probabilities of innovation and imitation.

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<sup>21</sup> Since  $\hat{Q}^\sigma = 1$ , it does not appear in the remaining equations.

Consider the case where the size of quality improvements,  $q$ , is sufficiently large relative to the North/South marginal costs of production ratio that a Northern firm can hold the world market with a single quality level improvement over a Southern copy.<sup>22</sup> Then the lead Northern firm chooses a price slightly less than the limit price,  $P_{ij}=qMC_2$ , thereby preventing sales of the Southern imitation of the previous lead good.

Free trade affects expected profits from innovation in two ways. On one hand, trade implies a larger market and thus greater demand for lead intermediate goods. On the other hand, innovators now face the possibility of losing their market to imitating Southern firms, as well as to innovating Northern firms. Thus, lead Northern firms have greater profits while in leadership positions, but will most likely face diminished tenure.<sup>23</sup>

If a Southern firm successfully imitates the lead product in its sector, it faces a marginal cost advantage relative to the Northern lead firm. Hence, it uses limit pricing to capture the entire world market for that good. Specifically, since the lowest price a Northern firm can charge is  $MC_1$ , a Southern imitator can charge slightly less than the limit price,  $P_2 = MC_1 = I$ , and still earn positive profits. As expected, demand for this good is greater with Southern production than with Northern production since the price of the good has dropped with imitation. Furthermore, successful Southern imitators face a greater flow of profits while producing the highest quality good under free trade than under autarky, since they are now selling to the world market.<sup>24</sup>

### Steady-State Probabilities of Innovation and Imitation with Trade

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<sup>22</sup> Suppose the lead Southern firm in a particular sector has imitated the lead Northern good. When the next Northern innovation occurs, one of two things will happen depending on  $MC_1/MC_2$ . If  $q > MC_1/MC_2 > 1$ , the Northern firm will set its price at  $\epsilon$  less than  $qMC_2$ , thereby undercutting all sales of the outmoded Southern copy. The Northern firm can do this since  $qMC_2$  is greater than its lowest possible price,  $MC_1$ . On the other hand, if  $MC_1/MC_2 > q$ , then the lead Northern firm can not under-price the Southern firm. In fact, the lead Northern firm will not even be able to sell its lead product since its new innovation would be more expensive in cost per quality units than the imitated second quality good. In that situation, the Northern firm would have to innovate twice before it could market its product.

<sup>23</sup> Since it is likely that imitation of the lead good occurs before the next innovation in a sector, both Northern leaders and followers will attempt innovation in certain situations as discussed in the appendix.

<sup>24</sup> When  $q > MC_1/MC_2$ , the lead Southern firm must imitate the lead Northern good in order to capture the world market. Hence, only another innovation can dislodge the lead Southern firm. If instead  $q < MC_1/MC_2$ , then a Southern firm could capture the market by imitating the good one quality level below the lead good. Thus, the lead Southern firm could lose its market either with the next Southern imitation or with the next Northern innovation, now two quality levels ahead of this firm's imitated product.

Following the methodology described in the autarky case, I use the expected flow of profits, along with the assumption of free entry in research, to derive the equilibrium probabilities of innovation and imitation in the North and the South, respectively:<sup>25</sup>

$$(2.1) \quad p_{11} = \left[ \frac{\mathbf{b}_c}{\mathbf{z}_{c2}(e^{-w} + 1)} \mathbf{W}_2 (MC_1 - MC_2) \right] - r_2,$$

$$p_{c2} = \frac{\frac{\mathbf{b}_1}{\mathbf{z}_{11}} (MC_2 - \frac{MC_1}{q}) \mathbf{W}_1 - \frac{\mathbf{b}_c}{\mathbf{z}_{c2}(e^{-w} + 1)} (MC_1 - MC_2) \mathbf{W}_2}{1 + r - \frac{\mathbf{b}_c}{\mathbf{z}_{c2}(e^{-w} + 1)} (MC_1 - MC_2) \mathbf{W}_2}, \quad \text{where}$$

$$\mathbf{W}_1 = \left[ L_1 A_1^{1/a} \left( \frac{MC_1}{MC_2} \right)^{1/a} + L_2 A_2^{1/a} \right] (1 - \mathbf{a})^{1/a},$$

$$\mathbf{W}_2 = \left[ L_1 A_1^{1/a} + L_2 A_2^{1/a} \left( \frac{MC_2}{MC_1} \right)^{1/\alpha} \right] (1 - \mathbf{a})^{1/a}, \quad \text{and } MC_1 = 1.$$

Since both countries grow at the same rate, driven by Northern technological progress, they face the same equilibrium world interest rate,  $r=r_1=r_2$ . Given this constant equilibrium interest rate, the above expressions also represent steady-state probabilities.

Notice that the steady-state probability of innovation depends positively on  $\mathbf{b}_c/\zeta_{c2}(e^{-w}+1)$ . Thus, the greater the spillover from imitation and the lower the cost of imitation, the greater the equilibrium probability of innovation. Hence, the more the South imports high technology goods, implying a high  $w$  and a low cost of imitation, the greater the probability of Northern innovation, all else equal. This reflects a pushing forward of Northern innovation when faced with competition from low cost imitation.

The probability of imitation depends negatively on the experience adjusted cost of Northern innovation. This implies that North-South competition also drives up rates of Southern imitation. The parameters for imitation however, enter ambiguously in equation (2.1). This reflects the fact that these parameters on the one hand, positively affect a

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<sup>25</sup> Note that  $\mathbf{J}_{C2k_j} = \mathbf{b}_c q^{k_j} = \mathbf{b}_c q^{k_{ij}-1}$  since the Southern quality level is exactly one level below that of the North when imitation is attempted. Furthermore,  $\mathbf{f}_c(k_{ij}-1)$  is defined as  $(1/\zeta_{c2}(e^{-w}+1)) q^{1-(k_{ij}/a)}$ , and  $\mathbf{f}_j(k_{ij})$  is again equal to  $(1/\zeta_{11}) q^{-k_{ij}/a}$ .

prospective imitator's probability of success, but on the other hand, decrease the potential imitator's expected tenure due to increased probability of further innovation.

Since at any one point in time each country may be using intermediate goods produced in different countries, final goods production is

$$Y_1 = A_1 L_1^a \sum_{j=1}^J [(q^{k_{1j}} x_{1k_j}) + (q^{k_{1j}} x_{2k_j}^*)]^{1-a}$$

$$Y_2 = A_2 L_2^a \sum_{j=1}^J [(q^{k_{2j}} x_{1k_j}^*) + (q^{k_{2j}} x_{2k_j})]^{1-a},$$

where  $0 < a < 1$  and asterisks denote imports. Substituting in implied demand and dividing through by labor yields an expression for the aggregate output per worker in each country:

$$\frac{Y_1}{L_1} = Q_1 A_1^{1/a} (1-\alpha)^{(1-a)/a} [D (\frac{MC_1}{qMC_2})^{(1-a)/a} + (1-D)]$$

$$\frac{Y_2}{L_2} = Q_2 A_2^{1/a} (1-\alpha)^{(1-a)/a} [D q^{(a-1)/a} + (1-D) (\frac{MC_2}{MC_1})^{(1-a)/a}],$$

where  $D$  is the proportion of intermediate good sectors with production in the North and  $(1-D)$  is the proportion of sectors with production in the South. Since aggregate output in both countries is increasing in  $Q_t$ , both countries will be growing at the same rate, equal to the growth rate of Northern technology.<sup>26</sup> Again, aggregate resources devoted to research and aggregate consumption in both countries are constant multiples of the Northern aggregate index of attained quality. Hence, setting the growth rate of  $Q_t$  (derived using the equilibrium probability of innovation) equal to the growth rate of consumption from household optimization yields an expression for the equilibrium world interest rate, assuming that both countries face the same  $r$  and  $q$ :

$$(2.2) \quad r^* = r + q \left[ \frac{b_c}{z_{c2}(e^{-w} + 1)} W_2 (MC_1 - MC_2) - r \right] / \left( \frac{1}{q^{(1-a)/a} - 1} + q \right).$$

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<sup>26</sup> For aggregate output growth to exactly equal Northern technology growth,  $D$  must be constant. I assume for simplicity that this is the case. This does not significantly affect the results since even if  $D$  were allowed to change over time, it would randomly fluctuate between values between zero and one.

To return to country growth rates, subtract  $r$  from equation (2.2) and divide by  $q$ :

$$(2.3) \mathbf{g}_1^* = \mathbf{g}_2^* = \left[ \frac{\mathbf{b}_c}{\mathbf{z}_{c2}(e^{-w} + I)} W_2 (MC_1 - MC_2) - r \right] / \left( \frac{1}{q^{(1-a)/a}} + q \right),$$

where  $MC_1 = 1$ . In both countries, the growth rate depends negatively on the cost of imitation. Hence, the higher the level of Southern imports of intermediate goods, the lower the cost of imitation and the greater the growth rate in both countries. However, the South can *not* simply lower its cost of imitation to zero, causing the growth rate of both countries to go to infinity. As the level of interaction,  $w$ , increases, there is a lower bound on the cost of imitation equal to  $\mathbf{z}_{c2}$  which effectively rules out such an outcome.

As previously mentioned, Northern firms enjoy higher profits while they are able to sell to the world market, but will face decreased expected tenure under free trade. Hence, there will be a positive or a negative feedback effect between innovation and imitation depending upon which of these two effects dominates. To see exactly what conditions determine the sign of the feedback effect, I compare the growth rate under free trade with imitation with the previous autarky growth rate. In so doing, I find that the free trade growth rate with imitative competition is greater than the Northern autarky growth rate if

$$(2.4) \frac{\mathbf{b}_c}{\mathbf{b}_1} \frac{\mathbf{z}_{11}}{\mathbf{z}_{c2}(e^{-w} + I)} > \frac{L_1 A_1^{1/a} MC_1 \left( \frac{q-1}{q} \right)}{\left( L_1 A_1^{1/a} + L_2 A_2^{1/a} \left( \frac{MC_2}{MC_1} \right)^{1/a} \right) (MC_1 - MC_2)}, \quad \text{where } MC_1 = 1.$$

The left-hand side of this expression shows that all else equal, the lower the experience-adjusted cost of imitation,  $\zeta_{c2}(e^{-w} + I)/\mathbf{b}_c$ , relative to the experience-adjusted cost of innovation,  $\zeta_{11}/\mathbf{b}_1$ , the more quickly Northern firms will innovate. Hence, if the cost of imitation is sufficiently low relative to that of innovation, then both countries grow more quickly under free trade with imitation than under autarky. This can be thought of in light of today's computer industry, where lead innovating firms know they will be quickly cloned and, hence, actually push their research forward more quickly when faced with strong imitative competition since that is the only way to stay in the market. The right-hand side of this expression shows, *ceteris paribus*, the larger the world market relative to

the Northern market, the more likely the Northern firms will increase the intensity of their research, thereby increasing the growth rate of both economies. This reflects the scale effects present in the demand for and, hence, the profitability of intermediate goods.

I previously assumed that the experience-adjusted cost of imitation is lower than the experience-adjusted cost of innovation. Furthermore, the size of the world market is obviously greater than the size of the Northern market alone. It is therefore likely that condition (2.4) holds and both countries face higher steady-state growth than under autarky. This result is similar to that of the Grossman and Helpman (1991a) varieties model of North-South trade and the inefficient followers case of the quality-ladder model. These results are also consistent with Feenstra's (1996) work stressing technological diffusion as the crucial element required for trade to lead to international convergence in growth rates. In this model, trade and technological diffusion go hand in hand since technology is embodied and there are no adjustment cost to using imported capital goods.

Still, even if growth rates are pushed up as the result of a positive feedback effect between innovation and imitation, the welfare effect is ambiguous for the North which bears the costs of increased rates of innovation and growth. On one hand, free trade reduces pricing distortions, but on the other hand, it further exacerbates rent destruction as increased competition forces faster innovation. From the Northern perspective, international property rights (IPRs) would be welfare enhancing since this would help remunerate Northern innovators for the positive spillover their research gives to Southern R&D. It is also interesting to note that if instead of leading to a positive feedback effect between innovation and imitation, free trade lead to a negative feedback effect, then it would also be in the South's best interest to impose IPRs. For example, suppose that some type of licensing arrangement between Northern innovators and Southern firms existed whereby Southern firms paid a licensing fee plus an adjustment cost which combined were still less than the cost of imitation. Then, it would be welfare (as well as growth) enhancing for both countries to enforce such a licensing arrangement.

It is more likely that some Southern sectors will give up imitation altogether, while other sectors will continue to imitate once North-South trade begins. This implies North-South divergence in aggregate technology. Still, in sectors where Southern imitation

continues, the North-South technology gap will remain approximately equal to zero. Which Southern sectors continue to imitate under free trade will depend on the past learning-to-learn in that sector prior to facing Northern competition. Nonetheless, the South will still grow with the North and, hence, conditional convergence still results.

## **V. Summary and Conclusion**

This paper presents an endogenous model of growth through technological progress, demonstrating both static and dynamic benefits for less developed countries when trading with developed countries. The concept of learning-to-learn in both imitative and innovative research is introduced, and a potential mechanism through which trade directly affects the process of imitation is explicitly modeled. Transitional dynamics and the determinants of comparative advantage in innovation are derived for the case of imitation without trade. This demonstrates positive dynamic effects for the South of technological diffusion through imitation. Finally, free trade with imitation generally leads to greater growth rates than imitation under autarky, implying a dynamic gain from trade for both the innovating and the imitating country.

This paper thus provides another argument, especially from the point of view of developing nations, in favor of free trade. In particular, these results suggest that developing nations should not fear that trade might cause their firms to specialize in imitation. This may be a temporary stage during which the developing country is catching up with developed nations. It may then take over the leadership position in certain sectors, or it may always remain the follower country. Nonetheless, the developing country will grow much more quickly both in the short and long run if it allows free trade and comparative advantage in innovation to determine whether any domestic industries will eventually take over leadership positions, rather than trying to isolate itself from the world in the hopes of forcing its industries to develop in a vacuum.

## Appendix

### Data for Figure 1.

Country	Symbol	Country	Symbol
Algeria	Al	Jamaica	Jm
Argentina.	Ar	Japan	J
Australia	Au	Korea, Rep.	K
Austria	At	Malaysia	Ma
Belgium-Luxembourg	B-L	Malta	Mt
Bolivia	Bo	Mexico	M
Brazil	Br	Morocco	Mo
Chile	Ch	Netherlands	N
Colombia	Co	New Zealand	NZ
Costa Rica	CR	Norway	No
Denmark	D	Pakistan	Pa
Ecuador	Ec	Peru	Pe
Egypt	Eg	Philippines	Ph
Finland	Fi	Portugal	Po
France-Monaco	F-M	Singapore	Si
Germany	G	Spain	S
Greece	Gr	Sri Lanka	Sr
Guatemala	Gu	Sweden	Sw
Honduras	Ho	Switzerland	St
Hungary	Hu	Tunisia	Tu
Iceland	Ic	Turkey	Tk
India	In	United Kingdom	UK
Iran	Ir	Uruguay	Ur
Ireland	Ire	Venezuela	V
Israel	Is	Yugoslavia	Y
Italy	It	Zaire	Z

Data on imports of capital goods from developed nations come from the U.N.'s *Commodity Trade Statistics* based on Standard International Trade Classes 7, 86, and 89 (SITC, Rev. 1), deflated using the U.S. Producer Price Index (PPI) for machinery and transport equipment, which are consolidated under the category of capital equipment for more recent years.<sup>27</sup> The proxy for imitative activity is based on international patent data given by the World Intellectual Property Organizations' *Industrial Property Statistics*, and is defined as the number of domestic patent applications by home residents minus U.S. patent applications by residents of that same country.<sup>28</sup> This is proxy generally underestimates the level of imitative activity in LDC's since in countries with little patent enforcement, few, including imitators, will bother to seek domestic patent protection.

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<sup>27</sup>Class 7 includes machinery and transport equipment; Class 86 includes instruments (optical, medical, and photographic), watches, and clocks; and Class 89 represents miscellaneous manufactured goods such as sound recorders, musical instruments, toys, and office supplies (including computers in later years).

<sup>28</sup> This assumes that the novelty requirements for patent grants are more lax in countries other than the United States. To the extent that this is in many cases inaccurate, this proxy is flawed. Furthermore, the imitation proxy also reflects patenting issues relating to enforcement of patent laws in the home country and in the U.S., as well as expected profits from sales at home versus in the U.S..

However, given the lack of observable measures of imitation, it is taken as a first attempt to measure national levels of imitation.<sup>29</sup>

### Decision to Undertake Research

**Under Autarky.** Comparing profits of leaders replacing themselves,  $\pi_{lead}$ , with profits of followers taking over the lead position,  $\pi_{fol}$ . Since the lead firm will now be selling a product twice as productive as its nearest competitor, it will be able to use a price slightly below  $q^2MC_i$ . However, it will also lose its profits from its previous innovation, which was priced slightly below  $qMC_i$ . Hence, the lead firm's incremental gain in profits is

$$\Delta\pi_{lead} = B [(q^2 - 1) q^{-1/a} q^{(k_{ij}+1)(1-a)/a} - (q-1) q^{k_{ij}(1-a)/a}], \quad \text{where}$$

$$B = L_i A_i^{1/\alpha} \{[(1-a)/q]^{1/a} MC_i\}.$$

On the other hand, a follower goes from having no profits to having positive profits when it takes over the lead position. Hence, a follower's incremental gain is

$$\Delta\pi_{fol} = B (q-1) q^{(k_{ij}+1)(1-a)/a}.$$

The gain to a follower firm is greater so long as  $q^{1/a} > 1$ , which is guaranteed by the assumptions that  $q > 1$  and  $0 < a < 1$ .

### Under Free Trade: Three Scenarios

Scenario 1: Neither the lead Northern  $k_{1j}$ th innovation nor the  $(k_{1j}-1)$ th innovation has yet been copied (i.e., the North-South technology gap is still large). Then the lead Northern firm's closest competitor is the Northern firm that had invented the  $(k_{1j}-1)$ th innovation. This situation yields the same limit pricing scheme and outcomes as in autarky except that  $B=(L_1A_1^{1/\alpha}+L_2A_2^{1/\alpha})\{[(1-a)/q]^{1/a}MC_1\}$ . Only followers will do research.

Scenario 2: The lead Northern  $k_{1j}$ th innovation has not yet been imitated, although the  $(k_{1j}-1)$ th innovation has been imitated. If the lead Northern firm replaces itself, it will now be selling a product twice as productive as its nearest competitor (a Southern imitator). Hence, it will use a limit price slightly below  $q^2MC_2$ . However, it will also lose its profits

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<sup>29</sup>Figure 1 considers the level of capital good imports from DCs since this should theoretically matter to imitative activity. For example, if a country imports a large volume of high technology capital goods, it is likely that the country is importing a large variety of capital goods thus increasing the number of goods which will potentially be targeted for imitation. Secondly, importing firms which distribute and service imported capital goods will gain greater knowledge of the goods which they import as the volume of goods which they service and repair increases. Thirdly, if we consider each person exposed to an imported good as having a certain probability of reverse engineering the good, then the scale of imports matters. Note however that this positive relationship remains statistically significant when imports are considered as a share of GDP. For a complete presentation of this empirical work see Connolly (1997).

from its previous innovation, which was sold at a price slightly below  $qMC_2$ . Thus, the lead firm's incremental gain in profits from replacing itself is

$$\Delta\pi_{lead} = D [(q^2MC_2 - MC_1) q^{-1/a} q^{(k_{ij}+1)(1-a)/a} - (qMC_2 - MC_1) q^{k_{ij}(1-a)/a}], \quad \text{where}$$

$$D = (L_1 A_1^{1/\alpha} (MC_1/MC_2)^{1/a} + L_2 A_2^{1/\alpha}) [(1-a)/q]^{1/a}.$$

When a follower replaces the lead firm, it sets its price so as to not only capture the market away from the ex-lead Northern firm, which is one quality level behind, but also to prevent the Southern imitating firm, now two quality levels behind, from reentering the market. Its limit price will be the smaller of  $qMC_1$  and  $q^2MC_2$ . Since I assume that  $q > MC_1/MC_2$ , the lower of the two is  $qMC_1$ . Therefore, a follower will sell its new innovation at a price slightly below  $qMC_1$  when it takes over the lead position. Hence, the incremental gain for the follower (with Southern firms at least two steps behind) is

$$\Delta\pi_{fol} = D (q - 1) MC_1 q^{(k_{ij}+1)(1-a)/a}, \quad \text{where } D \text{ is defined above.}$$

The incremental gain to a follower is greater than to the leader so long as  $q^{-1} + q^{1/a} - q^{(1-a)/a} > 1$ . Again, the assumptions that  $q > 1$  and  $0 < a < 1$  are sufficient to guarantee that this condition holds. Only followers will undertake innovative research.

Scenario 3: Imitation of the lead Northern innovation has already occurred. Then both leaders and followers face the same costs and incentives to bring about the next innovation. Hence, both leaders and followers will undertake innovative research.

### The Equilibrium Probability of Innovation under Autarky

Define the  $k_j$ th innovator as the firm which raises the quality level in intermediate good in sector  $j$ , from rung  $k_j - 1$  to  $k_j$ . The flow of profits earned by the  $k_j$ th innovator is

$$(A1) \quad p_{\Pi k_j} = (P_I - MC_1) x_{\Pi k_j} = (qMC_1 - MC_1) x_{\Pi k_j} = (q^{(a-1)/a} Y_I) q^{k_j(1-a)/a},$$

where  $Y_I = L_1 A_1^{1/a} (1-a)^{1/a} MC_1 (q-1)/q$ , and the subscript  $I$  denotes innovation.

Combining the probability density function for the time interval between the lead firm's innovation and the next innovation in that sector with the expression for  $p_{\Pi k_j}$  in equation (A1) yields an expression for the expected present value of profits for the  $k_j$ th innovation, calculated at  $t_{k_j}$ :

$$(A2) \quad E(v_{\Pi k_j}) = (q^{(a-1)/a} Y_I) q^{k_j(1-a)/a} / (r_1 + p_{\Pi k_j}).$$

Hence, the expected present value of profits (at the time of innovation) depends negatively on the interest rate,  $r_1$ , as well as the probability,  $p_{\Pi k_j}$ , that the  $(k_j + 1)$ th innovation will

occur, wiping out all of its current sales.<sup>30</sup> The equilibrium interest rate depends on the probability of innovation and hence, its equilibrium value is derived later.

The expected flow of net profits in the North from research by follower firms in industry  $j$ , with present quality level  $k_j$ , can then be written

$$(A3) \quad P_{IIk_j} = p_{IIk_j} E(v_{IIk_{j+1}}) - z_{IIk_j}.$$

This holds since the expected revenue from the  $(k_j+1)$ th innovation is  $p_{IIk_j} E(v_{IIk_{j+1}})$ , the probability of successful innovation times the expected present value of profits, given successful innovation. Finally, the total resource cost of innovative research in sector  $j$ ,  $z_{IIk_j}$ , must be subtracted from expected revenues so as to give the expected flow of net profits for the  $(k_j+1)$ th innovation.

If any innovative research is undertaken, free entry in research will guarantee that  $P_{IIk_j}=0$ . Substituting equation (A2) into (A3) and setting this expression equal to zero thereby yields the free entry condition:

$$(A4) \quad z_{IIk_j} = p_{IIk_j} [ Y_1 q^{k_j(1-a)/a} / (r_1 + p_{IIk_{j+1}}) ].$$

Here we see that resources devoted to R&D by followers in sector  $j$ , presently at quality rung  $k_j$ , depend positively on the present probability,  $p_{IIk_j}$ , of successful innovation for the  $(k_j+1)$ th quality good, but negatively on the probability of the successful innovation of the  $(k_j+2)$ th intermediate good which will be undertaken by the next period's followers. Once we substitute in equation (1.4) for  $p_{IIk_j}$  into equation (A4),  $z_{IIk_j}$  drops out, yielding an expression for the probability of successful innovation of the  $(k_j+2)$ th quality level:

$$p_{IIk_{j+1}} = f(k_{1j}) J_{1k_j} Y_1 q^{k_{1j}(1-a)/a} - r_1.$$

Since the North has been innovating, the highest quality level it has innovated in the past is equal to its current quality level, i.e.  $J_{1k_j} = b_1 q^{k_{1j}}$ . Substituting for  $J_{1k_j}$  and  $f(k_{1j})$ , we get the following expression for the probability of innovation:

$$(1.5) \quad p_{II} = \frac{b_1}{z_{II}} Y_1 - r_1.$$

### Market Value of Firms

The only firm with market value in any given sector is the lead firm since it is the only firm currently producing intermediate goods. Its market value is its expected present

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<sup>30</sup> To abstract from the ex-ante uncertainty faced by follower firms about whether or not they will be the first to succeed in bringing about the next innovation, I assume that research projects are supported by syndicates sufficiently large so as to diversify risk, but sufficiently small so as to not take into account their effects on the intermediate goods market. Since each research project is relatively small and has a purely idiosyncratic risk associated with it, this assumption implies that individual firms will concern themselves only with the expected present value of profits, rather than the randomness of the return.

value of profits for its innovation,  $E(v_{ik_j})$ , in equation (A2). Using the free entry condition in equation (A4) to find an expression for  $r_i + p_{ik_j}$ , and substituting this into  $E(v_{ik_j})$  yields

$$E(v_{ik_j}) = \frac{z_i}{b_i} q^{k_{ij}(a-1)/a}.$$

Thus, the value of the lead firm is simply the expected profits of its innovation, which coincides with the expected cost of bringing about the  $k_j$ th innovation. Aggregating across sectors, the aggregate market value of firms,  $V$ , is also a constant multiple of  $Q_i$ :

$$V = \sum_{j=1}^J E(v_{ik_j}) = \frac{z_i}{b_i} Q_i.$$

### Transitional Dynamics under Autarky

Let us define an intermediate goods sector specific budget constraint

$$y_{2k_a} = x_{2k_a} + c_{2k_a} + z_{C2k_a}, \quad \text{where}$$

$$y_{2k_a} = \frac{Y_2}{J} = q^{k_{2a}(1-a)/a} L_2,$$

$$x_{2k_a} = \frac{X_2}{J} = q^{k_{2a}(1-a)/a} L_2 \left( \frac{1-a}{q} \right),$$

$$c_{2k_a} = \frac{C_2}{J} = q^{k_{2a}(1-a)/a} \left\{ L_2 \left[ 1 - \left( \frac{1-a}{q} \right) \right] - p_{C2} \frac{z_{C2} \hat{Q}^s (e^{-w} + 1)}{b_C} \right\},$$

$$z_{C2k_a} = \frac{Z_{C2}}{J} = q^{k_{2a}(1-a)/a} p_{C2} \frac{z_{C2} \hat{Q}^s (e^{-w} + 1)}{b_C},$$

$$q^{k_{2a}(1-a)/a} = \frac{Q_2}{J}, \quad \text{and} \quad L_2 = L_2 A_2^{1/a} \left( \frac{1-a}{q} \right)^{(1-a)/a}.^{31}$$

As mentioned earlier, the growth rate of the aggregate quality level  $Q_2$  is

$$(A5) \quad \frac{\dot{Q}_2}{Q_2} = p_{C2k_a} (q^{(1-a)/a} - 1) = z_{C2k_a} f_C(k_a) J_{C2k_a} (q^{(1-a)/a} - 1).$$

Substituting for  $z_{C2k_a}$  using the sector specific budget constraint ( $z_{C2k_a} = y_{2k_a} - x_{2k_a} - c_{2k_a}$ ), equation (A5) yields

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<sup>31</sup>  $y_{2k_a}$  and  $c_{2k_a}$  respectively reflect aggregate final goods output and aggregate household consumption divided by the number of intermediate goods sectors,  $J$ . In other words, these do not reflect resources used by individual intermediate goods sectors, but rather are included, only to have an accurate measure of what resources are available for the average intermediate goods sector for research and production.

$$\begin{aligned}\frac{\dot{Q}_2}{Q_2} &= (y_{2k_a} - x_{2k_a} - c_{2k_a}) \mathbf{f}_C(k_a) \vartheta_{C2k_a} (q^{(1-a)/a} - 1) \\ &= (y_{2k_a} - x_{2k_a} - c_{2k_a}) \frac{\mathbf{b}_c}{\mathbf{z}_{C2} \hat{Q}^s (e^{-w} + 1)} q^{k_{2a}(a-1)/a} (q^{(1-a)/a} - 1).\end{aligned}$$

Substituting in for  $y_{2k_a}$  and  $x_{2k_a}$ , yields an expression for the growth rate of the Southern aggregate quality level, which is independent of the quality level  $k_j$ :

$$(A6) \quad \frac{\dot{Q}_2}{Q_2} = \{L_2[1 - (\frac{1-a}{q})] - c_2\} \frac{\mathbf{b}_c}{\mathbf{z}_{C2} \hat{Q}^s (e^{-w} + 1)} (q^{(1-a)/a} - 1),$$

where  $L_2 = L_2 A_2^{1/a} (\frac{1-a}{q})^{(1-a)/a}$  and  $c_2 = \frac{C_2}{Q_2}$ .<sup>32</sup> Since  $\hat{Q} = \frac{Q_2}{Q_1}$ , we know that

$$\frac{\dot{\hat{Q}}}{\hat{Q}} = \frac{\dot{Q}_2}{Q_2} - \frac{\dot{Q}_1}{Q_1} = \frac{\mathbf{b}_c}{\mathbf{z}_{C2} \hat{Q}^s (e^{-w} + 1)} (q^{(1-a)/a} - 1) \{L_2[1 - (\frac{1-a}{q})] - c_2\} - \gamma_1.$$

This is the first differential equation. Next, we want to derive the differential equation for  $C_2$ , which will be constant in steady-state. Since  $C_2 = C_2/Q_2$ , we know that  $\dot{C}_2/C_2 = (\dot{C}_2/C_2) - (\dot{Q}_2/Q_2)$ . We already have an expression for  $\dot{Q}_2/Q_2$ , so we now need an expression for  $\dot{C}_2/C_2$ . With no population growth,  $\dot{C}_2/C_2 = \dot{c}_2/c_2$ . Hence, substituting equation (1.15) for  $r_2$  into the growth rate of consumption in equation (1.12) yields

$$\begin{aligned}(A7) \quad \frac{\dot{C}_2}{C_2} &= \frac{1}{q} [s \frac{\dot{\hat{Q}}}{\hat{Q}} + (p_{C2k_a} p_{C2,k_a+1} / z_{C2k_a}) - p_{C2,k_a+1} - r] \\ &= \frac{1}{q} \left\{ \frac{\mathbf{b}_c}{\mathbf{z}_{C2} \hat{Q}^s (e^{-w} + 1)} [s (q^{(1-a)/a} - 1) (L_2 (1 - \frac{1-a}{q}) - c_2) + \right. \\ &\quad \left. L_2 q^{(1-a)/a} (\frac{1-a}{q}) (q-1) MC_2] - (s \gamma_1 + p_{C2,k_a+1} + r) \right\}.\end{aligned}$$

Subtracting equation (A6) from equation (A7) yields the second differential equation

$$\begin{aligned}\frac{\dot{c}_2}{c_2} &= \frac{\dot{C}_2}{C_2} - \frac{\dot{Q}_2}{Q_2} = \frac{\mathbf{b}_c}{\mathbf{z}_{C2} \hat{Q}^s (e^{-w} + 1)} \left\{ \left( \frac{s}{q} - 1 \right) (q^{(1-a)/a} - 1) \left( L_2 \left( 1 - \frac{1-a}{q} \right) - c_2 \right) + \right. \\ &\quad \left. \frac{L_2}{q} q^{(1-a)/a} \left( \frac{1-a}{q} \right) (q-1) MC_2 \right\} - \frac{1}{q} (s \gamma_1 + p_{C2,k_a+1} + r).\end{aligned}$$

<sup>32</sup> Note that  $\chi_2$  is used instead of  $\chi_{2a}$  in the above expression. It is possible to make this substitution since the two are equal for the average industry  $a$ .

## References

- Aghion, Philippe and Peter Howitt. "A Model of Growth Through Creative Destruction." *Econometrica*. 60, No. 2 (March 1992): 323-51.
- Barro, Robert and Xavier Sala-i-Martin. "Convergence across States and Regions." *Brookings Papers on Economic Activity*, No. 1 (1991): 107-82.
- . *Economic Growth*. New York: McGraw-Hill, Inc. (1995a).
- . "Technological Diffusion, Convergence, and Growth." NBER working paper No. 5151, June (1995b).
- Ben-David, Dan. "Trade and Convergence Among Countries." *Journal of International Economics*. 40 (1996): 279-98.
- Ben-David, Dan and A.K.M. Atiqur Rahman. "Technological Convergence and International Trade." working paper. University of Houston. (1996).
- Brezis, Elise, Paul Krugman, and Daniel Tsiddon (1993). "Leapfrogging in International Competition: A Theory of Cycles in National Technological Leadership." *American Economic Review*. Vol. 83, no. 5 (December 1993): 1211-19.
- Coe, David T. and Elhanan Helpman. "International R&D Spillovers." *European Economic Review*. Vol. 39, no. 5 (May 1995): 859-87.
- Coe, David T., Elhanan Helpman, and Alexander W. Hoffmaister. "North-South R&D Spillovers." NBER working paper No. 5048 (March 1995).
- Connolly, Michelle. "Technology, Trade and Growth: Some Empirical Findings." Duke University, Economics Department Working Paper, No. 97-34 (1997).
- Dollar, David. "Technological Innovation, Capital Mobility, and the Product Cycle in North-South Trade." *American Economic Review*. Vol. 76, No. 1 (March 1986): 177-90.
- Eaton, Jonathan and Samuel Kortum. "Trade in Ideas: Patenting and Productivity in the OECD." Ruth Pollak working paper No. 34 (February 1995).
- Evenson, Robert E. and Yoav Kislev. "A Stochastic Model of Applied Research." *Journal of Political Economy*. 84, 2 (April 1976): 265-81.
- Feenstra, Robert C. "Trade and Uneven Growth." *Journal of Development Economics*. 49 (1996): 229-256.

- Glass, Amy. "Product cycles and Market Penetration." *International Economic Review*. Forthcoming (1997).
- Grossman, Gene and Elhanan Helpman. *Innovation and Growth in the Global Economy*. Cambridge, Massachusetts: MIT Press, 1991a.
- . "Quality Ladders and Product Cycles." *Quarterly Journal of Economics*. May (1991b): 557-86.
- Keller, Wolfgang. "Are International R&D Spillovers Trade-Related? Analyzing Spillovers Among Randomly Matched Trade Partners." NBER working paper #6065 (June 1997).
- Krugman, Paul. "A Model of Innovation, Technology Transfer, and the World Distribution of Income." *Journal of Political Economy*. 87(April 1979): 253-266.
- Lesley, Robert. *History of the Portland Cement Industry in the United States*. Chicago, Illinois: International Trade Press, Inc., 1924.
- Mansfield, Edwin, Mark Swartz, and Samuel Wagner. "Imitation Costs and Patents: An Empirical Study." *Economic Journal*. 91 (December 1981): 907-918.
- Nelson, Richard R. and Edmund S. Phelps. "Investment in Humans, Technological Diffusion, and Economic Growth." *American Economic Review*. 56, No. 2 (May 1966), 69-75.
- Ohyama, Michihiro and Ronald W. Jones. "Technology Choice, Overtaking, and Comparative Advantage." *Review of International Economics*. Vol. 3, No. 2 (June 1995): 224-34.
- Rivera-Batiz, Luis A. and Paul Romer. "Economic Integration and Endogenous Growth." *Quarterly Journal of Economics*. 106, 2 (May 1991): 531-55.
- Romer, Paul. "Endogenous Technological Change." *Journal of Political Economy*. 98, no. 5 (1990): S71-S102.
- Sokoloff, Kenneth L. "Inventive Activity in Early Industrial America: Evidence from Patent Records, 1790-1846." *Journal of Economic History*. 48 (December 1988): 813-50.
- Stokey, Nancy. "Learning by Doing and the Introduction of New Goods." *Journal of Political Economy*. 96, No. 4 (August 1988): 701(17).
- Thompson, Ross. "Learning by Selling and Invention: The Case of the Sewing Machine." *Journal of Economic History*, 55 (May 1992): 285-307.

van Elkan, Rachel. "Catching Up and Slowing Down: Learning and Growth Patterns in an Open Economy." *Journal of International Economics*. 41 (August 1996): 95-111.

Young, Alwyn. "Learning By Doing and the Dynamic Effects of International Trade." *Quarterly Journal of Economics*. (1991): 369-405.