

Piecewise Procurement of a Large-Scale Project

Huseyin Yildirim*

Department of Economics,

Duke University

Durham NC 27708

E-mail: yildirh@econ.duke.edu

Phone: (919) 660-1805

First Version: September 2002

This version: December 2003

Abstract

This paper studies the optimal procurement of a large-scale project. The project consists of several subprojects to be procured in a predetermined order and yields its full return upon completion. In the unique Markov perfect equilibrium of the dynamic procurement game, I find that: (1) Even though the procurer lacks long term commitment power, both the procurer and suppliers strictly prefer the project to move forward; (2) unlike the static setting, the procurer's optimal strategy depends on the number of suppliers and more importantly, it is nonmonotonic. As one more supplier participates in the procurement auction, the procurer softens competition in the initial stages by including more cost "types" while increasing competition in the mature stages, (3) this, in turn, implies that existing suppliers might favor participation of additional suppliers, and actively search for new bidders along with the procurer, and (4) absent the scheduling and resource constraints, the procurer would prefer to procure the project as one piece if the suppliers' technology exhibited constant or increasing returns, and no additional suppliers were attracted to bid. This rationalizes the frequent division of large-scale projects on the grounds of either decreasing returns to scale technology, e.g., capacity constraints, or enabling smaller firms to enter the bidding process. I also demonstrate that the optimal dynamic mechanism can be easily implemented via a sequence of dominant strategy auctions of the Vickrey type.

Key Words: Sequential Procurement, Endogenous Valuation, Markov Perfect Equilibrium, Project Division

JEL Classification: C73, D44, L21

1 Introduction

Understanding the nature of government procurement processes has significant potential benefits as government procurements of goods and services in many countries constitute

*I thank Jim Anton, Wolfgang Kohler, Tracy Lewis, Curt Taylor for their comments and suggestions. I also thank Gary Biglaiser, Guiseppe Lopomo, and Tom Nechyba for valuable discussions. Comments from the seminar participants at University of Virginia and 2003 Midwest Theory Conference are also greatly appreciated. All errors are mine.

more than 10 percent of national income[Hoekman and Mavroidis (1997), McAfee and McMillan (1988)]. Governments and public agencies contract out a variety of large-scale projects to private firms such as building road networks, construction of schools and multi-facility sport complexes, and renovation and repair of multi-building historic sites and university campuses, as well as a variety of small-scale projects such as snow clearing, garbage collection, and conducting opinion polls. This paper studies the optimal procurement of a large-scale project through competitive bidding.¹

Unlike small-scale projects, large-scale projects often times need to be divided into small pieces or subprojects and procured in a predetermined order. Examples abound. Most of the major reconstruction and modernization projects in developing countries, which are supported by the World Bank consist of subprojects.² Similarly, schools periodically undergo major repair and renovation work including multiple buildings. There are several institutional and practical reasons for procuring a large-scale project in a piecemeal fashion: First, current governments may not commit resources that are to be available to future governments. Second, there may arise significant scheduling constraints in undertaking large-scale projects like road networks and repair of college buildings on campuses, which require shutting down essential activities. Third, there may be too few firms with sufficient resources to undertake the entire project. This is especially true in the World Bank projects for which the Bank preferably relies on domestic firms in developing countries to revive their economies. Fourth, but not the last, supply technologies may exhibit decreasing returns to scale due to, e.g., capacity constraints.

How would the procurer optimally design a procurement auction for each subproject, especially when she cannot commit to the terms of future trades? Would firms facilitate the progress of the project by offering price discounts early on, knowing that doing so will leave fewer profit opportunities? Would the procurer and, more interestingly, the existing firms actively search for new firms to bid for the project? Absent the concerns alluded to above, what would the optimal division of a large-scale project be from the procurer's perspective?

To address these and other related issues, I construct a simple dynamic procurement model in which the procurer owns a multi-stage project. Each stage corresponds to a

¹While primary focus of this paper is on public contracts, similar issues discussed in this paper arise in private sector procurement as well.

²After the war, the Federation of Bosnia and Herzegovina with the support of the World Bank launched a major reconstruction and recovery project in 1997. The project has been divided into more than 400 subprojects scheduled according to their emergency. Moreover, subprojects have been procured through competitive bidding with the Bank's oversight. For more on this project, visit <http://www.lora.com.ba/lora/news.htm>.

subproject. There are N firms, each of which is capable of doing all subprojects (possibly through their own network of subcontractors). I assume firms' current costs are privately known and vary over time. These costs can reflect, for instance, firms' labor, fuel and raw material costs, or simply their opportunity costs fluctuating with market conditions.³ In each period, the procurer designs an optimal procurement auction for the current subproject that determines the payment and the allocation functions based on firms' cost reports. I assume that the procurer has limited commitment power and thus can only commit to the terms of the current auction.⁴ I also assume that both the procurer and firms base their strategies only on the state of the project. Thus, the Markov Perfect Equilibrium (MPE) concept is used throughout. While this rules out the interesting aspects of reputation building and collusion, it serves the purpose of the paper, as I wish to investigate the effects of the physical progress of the project on parties' relationship.

The complicated interaction between the procurer and firms can be simplified by using dynamic programming arguments. In Section 2, I show the existence of a unique MPE in which the procurer follows a cutoff strategy and, like in a static (one-period) setting, selects the lowest cost supplier for each task (by paying, of course, appropriate rents). Unlike the static setting though, the cutoff in the dynamic setting is endogenously determined for each task. I find that the procurer increases the cutoff with the progress of the project, and that not surprisingly, she strictly prefers the project to move forward. Somewhat surprisingly however, I also find that firms too prefer the project to move forward by early discounts, despite the fact that this will leave fewer profit opportunities, and that they sometimes incur losses for which the procurer cannot commit (even implicitly) to compensate in the future. The intuition is that by increasing the cutoff and thus including more cost "types" along with the progress of the project, the procurer enables firms to earn *progressively* greater information rents that become the "prizes" to win.

As for any optimal mechanism analysis, the practical value, of course, comes from its implementation through some well-known auctions. In Section 2, I demonstrate that the dynamic procurement mechanism identified in this paper can be easily implemented via a

³I assume that winning the current subproject does not give an advantage due to, say learning-by-doing, or a disadvantage due to, say, capacity constraints to the winner in future biddings. While I certainly acknowledge the importance of these effects, here I focus on projects with sufficiently routine tasks and intend to highlight other dynamic effects, which, I believe, will be present in models with these features. Nonetheless, I briefly discuss how the results might change with learning-by-doing.

⁴Government procurement agents usually face legal and administrative constraints preventing them from committing resources for their successors.

sequence of dominant strategy auctions of the Vickrey type.

Next, I investigate two comparative statics in Section 3. The first one deals with how the participation of one more firm to the competition affects the procurement design and parties' expected value of the project. Unlike the static setting in which the cutoff is independent of the number of bidders [e.g., Laffont and Tirole (1987), McAfee and McMillan (1987), Riordan and Sappington (1987)], the procurer modifies her optimal strategy in the dynamic setting in response to one more bidder. This is because the additional firm changes the expectations about the progress of the project. Interestingly, the procurer *increases* the cutoff with one more firm in the initial stages of the project and reduces it in mature stages, implying that there is a non-monotonic relationship between the number of bidders and the procurer's optimal strategy. Intuitively, by increasing the cutoff early on, the procurer softens the competition to ensure the progress of the project. From the existing firms' point of view, this implies that an additional firm has two opposing effects on their expected profits: First, all else equal, an additional firm reduces the chances of being the winner, but second, it also leads the procurer to raise the cutoff, and thereby leave greater information rents. I show that if there are sufficiently many subprojects ahead, the increase in rents can more than offset the negative competition effect, and thus existing firms might indeed *favor* more participation into the procurement competition. An interesting implication of this finding is that for large projects, existing firms might actively search for new bidders along with the procurer. This observation is important because Bulow and Klemperer (1996) argue that attracting a new bidder is more valuable for the procurer (the seller in their setup) than having more bargaining power. Another implication is that a large firm might prefer to enter the procurement auction by splitting into two or more independent entities. Despite the nonmonotonicity in the optimal strategy, the procurer always prefers more participation into the procurement.

The second comparative static in Section 3 asks whether the buyer likes to deal with long- or short-sighted firms. While long-sighted firms with high discount factors are willing to offer severe price discounts in the initial states of the project to facilitate its progress, they tend to raise their prices in states near completion when the buyer's value of the project is high. Thus, I find that the procurer is better off dealing with long-sighted firms only if the project is sufficiently large in order for initial price discounts to overcome future price increases. Otherwise, small-scale projects are supplied at a lower expected cost by

short-sighted suppliers.

Absent the resource and scheduling constraints alluded to above, an important control variable for the procurer is to divide a large-scale project into subprojects, if doing so would be optimal. In Section 5, I identify two opposing incentives for the procurer to divide the project: First, to do so, the procurer takes advantage of repeated competition, but second the procurer suffers from further delay in completing the project. I then show, under very general conditions, that if the production technology exhibits constant or increasing returns to scale, and no new (possibly smaller) supplier is attracted to bid, then the procurer is better off procuring the project as one piece. This finding rationalizes the frequent division of large-scale projects to either reduce the procurement cost when facing a decreasing returns to scale technology or attract smaller firms into competition.

The rest of the paper is organized as follows. In Section 4, I analyze the complete information setup as a benchmark, in which firms' current costs are common knowledge. In Section 5, I check the robustness of my findings by extending the model to a Laffont and Tirole (1993) framework where firms can exert effort to reduce their supply costs. Finally, concluding remarks are made in Section 6.

Related Work Papers by Laffont and Tirole (1987), Manelli and Vincent (1995), McAfee and McMillan (1987), and Riordan and Sappington (1987) investigate optimal procurement auctions for one-time lump-sum purchases only. Thus, repeated competition along with the progress of the project does not arise. Gale, Hausch, and Stegeman (2000) study the possibility of subcontracting between suppliers when buyers are non-strategic. Laffont and Tirole (1988), and Luton and McAfee (1986) consider sequential auctions for the repeat purchases of the same good. Unlike the present work, they assume the procurer can commit to a long-term contract. Lewis and Yildirim (2002a, b) focus on a repeated procurement model in which the same good is purchased and, much like in the present setting, the procurer cannot commit to long-term contracts. However, they focus on the effects of learning by doing on the part of suppliers, and do not consider large-scale projects.⁵

Anton and Yao (1987), and Rob (1986) also address the question of how a procuring agency, e.g., the Department of Defense should procure a large-scale project, e.g., a major weapon system, by optimally dividing the project to better exploit competition. Specifically,

⁵Papers by Lewis (1983) and Krishna (1993) examine a sequential procurement competition between the incumbent and potential entrants. Although their focus is different, they also recognize how firms' valuations of each unit of good are affected by the anticipation of the future auctions.

these papers consider a sequential acquisition process in which a fraction of the project is first awarded to a sole source who learns by doing, and then the rest is opened to competition by transferring the technology, though imperfectly, to a second source. While both papers examine the optimal staging of a large project when there is learning by doing in the first stage, there are two main differences: First, I abstract from learning aspect and focus, instead, on the continuing competition among same firms along with the progress of the project. Second, I allow for more than two stages, and this is important to some of the conclusions.

Perhaps, Lewis (1986) is closest paper in spirit to the present work. Like here, Lewis also presents a piecewise procurement model of large-scale projects. However, his focus is on rationalizing the frequent cost overruns in a long-term bilateral relationship between a procurer and a supplier, where neither party has long-term commitment ability and the supplier has better information about the type of the project than does the procurer. He shows that in equilibrium, the funding “standard” declines over time, which makes the cancellation of the project less likely as it progresses. While this scheme gives the supplier an incentive to work hard early on to build a reputation and guarantee future funding, it also leads the supplier to raise future prices, once the procurer is “locked” into the project. Although such a reputation building is not possible in the current analysis with multiple suppliers, like in Lewis, the buyer’s value of the project increases and thus she becomes more willing to make higher payments as the project nears completion. To reap these high payments in future periods, suppliers sometimes end up incurring losses early on, which somewhat resembles the reputation building in Lewis.

This paper also relates to the literature on sequential auctions where a seller repeatedly auctions off single- or multi-units of the same good. Papers by Ashenfelter (1989), Bernhardt and Scoones (1994), Caillaud and Mezzetti (2003), Gale and Hausch (1994), McAfee and Vincent (1993), and Jeitschko (1999), among others, consider sequential sales of multiple units of the same good. Unlike the present setting, the auctioneer in these papers values all units the same so that no complementarity between units arises. McAfee and Vincent (1997) analyzes a case where the seller lacks commitment not to re-auction the same unit if it is not sold in previous periods. While the procurer can re-auction the same subproject if the cut-off cost is exceeded in my setting too, unlike McAfee and Vincent, costs are drawn anew in each period and there are multiple subprojects to be auctioned off.

2 A Model of Sequential Procurement

There are $N + 1$ risk-neutral agents in the model: one buyer (she) and N suppliers. The buyer owns a multi-stage project, e.g., a multi-stage road network, a multi-facility sport complex, or a multi-building campus renovation project.⁶ The project consists of ρ^u stages or subprojects to be completed in a pre-determined order, where ρ^u is a natural number. The buyer receives a value $V > 0$ once the project is completed. While completing each subproject might yield some intermediate payoff, for simplicity, I normalize these payoffs to zero to better focus on the dynamics of the procurement.⁷

Let ρ be the state variable indicating that first $(\rho - 1)$ subprojects have been completed and subproject ρ is being auctioned off. While each supplier is capable of completing all subprojects⁸, the buyer auctions off each subproject that takes only one period to complete.⁹ I assume that the procurer has limited commitment power. In particular, I assume the procurer can only commit to the terms of the current procurement contract, and offers a (possibly) new contract in future periods. Each supplier privately draws a cost of undertaking the subproject, c_i , in each period from a twice differentiable distribution function, $F(c_i)$, which is independently and identically distributed (IID) over time and across suppliers.¹⁰ The support of distribution is $[0, \bar{c}]$ with $\bar{c} > 0$ and $F'(c_i) = f(c_i) > 0$. For future reference, let $h(c_i) \equiv c_i + \frac{F(c_i)}{f(c_i)}$ denote the “virtual” cost entailing the cost of undertaking the task plus a term for information rents. As is standard in the procurement literature, I ensure

⁶In what follows, I use the procurer and buyer interchangeably.

⁷To be more specific, one can let v_ρ be the cumulative payoff of completing the first ρ subprojects, where v_ρ is nondecreasing and $v_{\rho^u} \equiv V > v_{\rho^u - 1}$. If the marginal return of each subproject is nondecreasing, i.e., $v_\rho - v_{\rho-1} \geq v_{\rho-1} - v_{\rho-2}$, the results will go through. Note that such a “convex” cumulative payoff property is likely to hold in most large-scale projects for which each completed subproject increases the payoff from the previous ones. A road network project would be a good example. For simplicity, I set $v_\rho = 0$ for $\rho < \rho^u$.

⁸This should be interpreted in a broader sense. In particular, suppliers themselves might have several subcontractors capable of doing different parts of the project.

⁹In general, one can assume the completion time to be $t \geq 1$ periods. This can be incorporated into the analysis without changing the main point of the paper.

¹⁰The IID assumption rules out the strategic learning and signalling aspect among suppliers, which might be potentially interesting. However, I make the IID assumption for two reasons: First, it helps isolate the impact of the progress of the project on agents’ relationship without clouding the model with additional features. Second, it simplifies the analysis greatly, especially since the procurer lacks long-term commitment in my model. See, for instance, Laffont and Tirole (1993, ch 9) for a review and difficulties of repeated procurement with correlated costs in bilateral relationships between the procurer and a single supplier. I should probably note that when suppliers’ costs are correlated (even slightly), the procurer can optimally design an auction a la Cremer and McLean (1988) and achieve the first-best described in Section 4, though such a mechanism requires rather strong assumptions such as the ability of imposing large penalties on suppliers in case their cost reports do not match.

the virtual cost is nondecreasing by making

$$\text{Assumption 1 : } \frac{d}{dc_i} \left[\frac{F(c_i)}{f(c_i)} \right] \geq 0.$$

The assumption that firms' costs are distributed IID over time deserves further comment. In reality, a firm's cost of undertaking a subproject would have (at least) two components: One task specific component capturing the unanticipated difficulties of the task ahead and one market specific component capturing the unanticipated volatility of input prices used in production and/or of the firm's outside opportunities. While it is reasonable to assume that the task specific cost would remain the same over time until the subproject in question is completed, the market specific cost can change over time even for the same subproject. Which type of cost is more important in shaping the procurer's and firms' strategies will surely depend on the type of the large-scale project in hand. By focusing on the fluctuations in market specific costs only, I am implicitly assuming a relatively well-defined project with routine tasks whose levels of difficulties can be ascertained in advance. This seems consistent with the projects in developing countries in which input markets are not well-developed and therefore input prices are likely to be less certain than the job itself.¹¹ The same would not be true however, if the project consisted of relatively innovative tasks that have not been undertaken by the firms before.

The interaction between the buyer and suppliers is modeled as an infinite horizon Markov game where parties base their strategies only on the state of the project (and on current costs for suppliers) and Markov Perfect Equilibrium (MPE) is the equilibrium concept throughout the paper. To be precise, the timing and information structure of this dynamic procurement model is as follows:

- In each period, upon observing the state (ρ) of the project, the procurer offers a contract $\{P_i(\rho, \hat{\mathbf{c}}), \lambda_i(\rho, \hat{\mathbf{c}})\}$ to supplier i , where $P_i(\rho, \hat{\mathbf{c}})$ and $\lambda_i(\rho, \hat{\mathbf{c}})$ denote the payment and the probability of awarding the subproject based on the state of the project and suppliers' cost reports, $\hat{\mathbf{c}} \equiv (\hat{c}_1, \dots, \hat{c}_N)$.
- Each supplier privately observes its current cost and decides whether or not to par-

¹¹While it would be desirable to incorporate both types of cost uncertainties, it will also make the model significantly less tractable. Consider, for instance, the other extreme where the only uncertainty is about the difficulty of the tasks ahead. Again, assume that the cost of each task is privately known by each supplier. Auctioning of each task in this case will call for Coasian type dynamics characterized in McAfee and Vincent (1988) in addition to the dynamics highlighted below, because the private information is persistent within a given subproject.

ticipate in the current procurement auction. (In equilibrium, all suppliers agree to participate.) Suppliers' outside opportunities in each period are normalized to 0.

- Finally, suppliers *simultaneously* report their current costs to the buyer. If one is awarded the task, then the project moves to the next task ($\rho + 1$). However, it is possible that the buyer may find costs too high and decide to wait for one period.¹² This ongoing procurement process starts next period with the revised state and with the exception that suppliers draw new costs.

Let $B(\rho)$ and $S_i(\rho)$ be the expected present value for the buyer and for supplier i respectively from participating in the current and future procurements given the current state, ρ . All parties discount future returns and costs by $\delta \in (0, 1)$.

Due to the Revelation Principle, e.g., Myerson (1979), when determining the optimal current contract, the procurer can restrict attention to truth-telling direct mechanisms.¹³ Thus, it suffices for her to take two constraints into account for each supplier: First, the contract must induce supplier i to truthfully report his cost, c_i , given that others also do so. That is, truth-telling must be a Bayesian-Nash equilibrium for suppliers. More formally, the contract $\{P_i(\rho, \hat{\mathbf{c}}), \lambda_i(\rho, \hat{\mathbf{c}})\}$ must satisfy the following dynamic program for supplier i 's cost report:

$$S_i(\rho, c_i) = \max_{\hat{c}_i} S_i(\rho, \hat{c}_i) \equiv P_i(\rho, \hat{c}_i) - \lambda_i(\rho, \hat{c}_i)c_i + \delta S_i(\rho) + \delta \sum_{k=1}^N \lambda_k(\rho, \hat{c}_i) \Delta S_i(\rho) \quad (\text{IC}_i)$$

where $S_i(\rho, c_i) \equiv E_{c_{-i}} [S_i(\rho, \mathbf{c})]$, $S_i(\rho) \equiv E_{c_i} [S_i(\rho, c_i)]$, $P_i(\rho, c_i) \equiv E_{c_{-i}} [P_i(\rho, \mathbf{c})]$, and $\lambda_i(\rho, c_i) \equiv E_{c_{-i}} [\lambda_i(\rho, \mathbf{c})]$ with E_c being the expectation operator with respect to c , and $\mathbf{c}_{-i} \equiv (c_1, \dots, c_{i-1}, c_{i+1}, \dots, c_N)$. The Δ is the usual difference operator between states ($\rho + 1$) and ρ throughout.

Eq (IC _{i}) indicates that supplier i decides on his cost report by maximizing his discounted expected value in the relationship. This value is comprised of the expected current and future profits. The current profit is the expected payment, $P_i(\rho, \hat{c}_i)$, minus the expected cost of undertaking the subproject, $\lambda_i(\rho, \hat{c}_i)c_i$. The future expected returns are the

¹²Note that the same subproject may be put up for bids if it fails to be awarded in the previous periods. See, for example, McAfee and Vincent (1997) on a similar point with one object.

¹³Note that in general when suppliers' costs are correlated over time and the procurer cannot commit to a long-term contract for the entire relationship, the Revelation Principle in its "standard" form does not apply. In particular, suppliers become reluctant to reveal their types early on, and thus substantial pooling in equilibrium may occur [e.g., Laffont and Tirole (1993), ch. 9]. However, since costs are assumed to be IID over time in my model, current costs do not reveal any information about future costs, and thus the standard Revelation Principle can be used.

discounted expected surplus, $\delta S_i(\rho)$, and the increase in the expected continuation value, $\delta \sum_{k=1}^N \lambda_k(\rho, \widehat{c}_i) \Delta S_i(\rho)$ when the subproject is assigned to a supplier. It is important to note that when finding the continuation value, supplier i considers the probability that the project will be assigned to *some* supplier, not just to himself. As we will see below, this will imply that the winning supplier will exert a positive externality on others by moving the project forward. Since the optimal contract induces truthful reporting for supplier i , it must satisfy the following condition, which is derived by applying the Envelope Theorem on (IC_i) :

$$\frac{dS_i(\rho, c_i)}{dc_i} = -\lambda_i(\rho, c_i) \quad (1)$$

That is, supplier i 's expected discounted profits decrease in his current cost at the rate of his diminished probability of winning.

Second, the contract must yield supplier i expected discounted returns at least as high as his expected returns when he rejects the current contract. That is, the contract must satisfy the following participation constraint:

$$S_i(\rho, c_i) \geq \delta R_i(\rho) \quad (IR_i)$$

where $R_i(\rho)$ is the expected net discounted value if supplier i rejects the current contract.

For now, I do not make any assumption regarding $R_i(\rho)$, as some results turn out to be independent of this value.

Combining (IC_i) , (IR_i) , and (1), the following lemma characterizes the implementable contracts.

Lemma 1. *For any procurement allocation satisfying (IC) and (IR) , $\lambda_i(\rho, c_i)$ is nonincreasing in c_i , and the payment and suppliers' surplus are given respectively by:*

$$P_i(\rho, c_i) = \delta[R_i(\rho) - S_i(\rho)] + \lambda_i(\rho, c_i)c_i + \int_{c_i}^{\bar{c}} \lambda_i(\rho, \tilde{c}_i) d\tilde{c}_i - \delta \sum_{k=1}^N \lambda_k(\rho, \widehat{c}_i) \Delta S_i(\rho) \quad (2)$$

$$S_i(\rho) = \delta R_i(\rho) + E_{c_i} \left[\lambda_i(\rho, c_i) \frac{F(c_i)}{f(c_i)} \right] \quad (3)$$

Proof. All proofs are relegated to an appendix. ■

The intuition behind (2) and (3) will be more transparent below, once the equilibrium $R_i(\rho)$ is characterized.

Having described the suppliers' behavior, I now specify the procurer's program and determine her optimal strategy. Note that once the project is completed, the buyer optimally

severs the relationship with suppliers and enjoys the benefit V . This implies the following boundary conditions: $B(\rho^u + 1) = V$ and $S_i(\rho^u + 1) = 0$. In the previous stages of the project however, the buyer solves the following dynamic program to determine the optimal current contract:

$$B(\rho) = \max_{\{\lambda_i(\cdot), P_i(\cdot)\}} \delta B(\rho) + E_c \sum_{i=1}^N [-P_i(\rho, \mathbf{c}) + \delta \lambda_i(\rho, \mathbf{c}) \Delta B(\rho)]$$

subject to (IR_i) and (IC_i) for all i . (4)

According to (4), the buyer maximizes her expected present value in the relationship by choosing the contract $\{P_i(\rho, \mathbf{c}), \lambda_i(\rho, \mathbf{c})\}$ for supplier i in state ρ . This value consists of the discounted expected value, $\delta B(\rho)$, and the current benefit, normalized to 0, minus payments to the suppliers, and plus the increase in the discounted expected future value when the project is assigned to firm i , $\delta \lambda_i(\rho, \mathbf{c}) \Delta B(\rho)$.

Inserting payments from (2) into (4) reduces the buyer's program to:

$$(1 - \delta)B(\rho) = \max_{\{\lambda_i(\cdot)\}} \sum_{i=1}^N E_c \lambda_i(\rho, \mathbf{c}) [-h(c_i) + \delta \Delta W(\rho)] - \sum_{i=1}^N \delta [R_i(\rho) - S_i(\rho)] \quad (5)$$

where we define the total surplus as $W(\rho) \equiv B(\rho) + \sum_{k=1}^N S_k(\rho)$.

It is clear from (5) that the procurer adopts the following selection strategy:

$$\lambda_i(\rho, c) = \begin{cases} 1, & \text{if } c_i \leq \min \{c(\rho), \min_{j \neq i} \{c_j\}\} \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

where we define the cut-off $c(\rho)$ such that $h(c(\rho)) \equiv \delta \Delta W(\rho)$.

According to (6), the buyer awards subproject ρ to the lowest-cost supplier unless this cost is too high. Note that whenever the subproject is assigned to a supplier, it is assigned in a socially efficient way. This is of course due to the IID cost assumption in that suppliers remain ex-ante symmetric with respect to the future subprojects.¹⁴ Thus, the optimal strategy is similar to that in a one-period model. Yet, there are two important differences: First, the cutpoint is now endogenously determined in each stage and increases as the project matures. Second, unlike a one-period model, the cutpoint is a function of number of suppliers. These remarks will become clearer in the ensuing analysis.

¹⁴In particular, if winning the current subproject created a systematic efficiency gap between the winner and losers because of, say, learning-by-doing, then the procurer would sometimes optimally favor the inefficient supplier to balance the future competition. This is the standard "handicapping" result in optimal auctions with unevenly matched bidders. See, for instance, Fudenberg and Tirole (1991, ch.7) for a static setting, and Lewis and Yildirim (2002b) for a dynamic setting.

Determining the unique equilibrium of this dynamic procurement game amounts to determining the unique sequence of $\{c(\rho)\}$. Let $G(c; N) \equiv 1 - [1 - F(c)]^N$ be the distribution function of $\min\{c_1, \dots, c_N\}$ and define the following function:

$$\Phi(x; \delta) = \frac{1 - \delta}{\delta} h(x) + \frac{F(x)}{f(x)} G(x; N) + \int_0^x G(c; N) dc, \quad \text{for } x > 0. \quad (7)$$

Lemma 2.

- a) $\Phi(0; \delta) = 0$, $\Phi'(x; \delta) > 0$ for all $x > 0$, and $\Phi'(0; \delta) = \frac{1-\delta}{\delta} h'(0) \geq 0$.
- b) For $\rho \leq \rho^u$, $\overline{W}(\rho + 1) = \Phi(c(\rho); \delta)$, $\overline{W}(\rho) = \Phi(c(\rho); 1)$ where $\overline{W}(\rho) = (1 - \delta)W(\rho)$.

Lemma 2 provides the properties of Φ , which will be important in proving our results and it shows how the backward induction works to find $\{c(\rho)\}$. Figure 1 further demonstrates these findings in a graph. For the clarity of exposition below, I assume that the upper bound of cost distribution is large enough that the procurer finds it optimal to exclude some cost types in each state.¹⁵ More formally, I make

$$\text{Assumption 2: } (1 - \delta)V < \Phi(\bar{c}; \delta)$$

The following proposition characterizes the equilibrium:

Proposition 1: *There exists a unique MPE of the dynamic procurement game, and it has these properties: For $\rho \leq \rho^u$,*

- a) $c(\rho) > 0$ and $c(\rho)$ is increasing in ρ .
- b) Total surplus, $W(\rho)$, increases at an increasing rate as the project moves forward.

That is, $\Delta W(\rho) > 0$ and $\Delta W(\rho) > \Delta W(\rho - 1)$.

According to Proposition 1, the buyer chooses a higher cutpoint as the project moves forward. This implies that the average waiting time in state ρ given by $w(\rho) = \frac{1}{G(c(\rho); N)}$, shrinks with the progress of the project. Thus, even though subprojects are *ex-ante* identical, on average, initial subprojects are completed in relatively longer periods than the later ones. By Lemma 2, the total surplus is given by $W(\rho) = \frac{1}{1-\delta} \Phi(c(\rho); 1)$. Since I have not placed any assumption on $R_i(\rho)$, Proposition 1 reveals that the total surplus is independent of this value. In other words, whatever threat the procurer uses on the supplier who refuses to participate, the total surplus generated in the relationship remains unchanged. This is simply because the threat only affects nonparticipating suppliers' (off-equilibrium) continuation value, which

¹⁵Otherwise, I simply need to keep track of uninteresting cases in which $c(\rho) = \bar{c}$, and in equilibrium, the task is assigned to a supplier with certainty.

in turn only affects the payment. Since the procurer's demand is inelastic by definition, the procurer's threat determines only how parties share the surplus. In equilibrium, part (b) of Proposition 1 indicates that the total surplus to be shared increases as the project moves forward. Hence, the optimal threat for the buyer is to exclude all suppliers from the current procurement auction if one of them refuses to participate.¹⁶ Formally, in what follows, I set

$$R_i(\rho) = S_i(\rho) \quad (8)$$

The fact that the total surplus increases at an increasing rate in part (b) will be very useful in determining the optimal division of the project in Section 5.1.

In light of Proposition 1, it is intuitive that because the procurer chooses the terms of the procurement contracts, she should be better off with the progress of the project. Yet, it is not clear if the same is true for suppliers given that the progress means fewer profit opportunities. To determine parties' present values of the project in state ρ , I use (5), (6), and (8) to find:

$$B(\rho) = \frac{1}{1-\delta} \int_0^{c(\rho)} h'(c)G(c; N)dc \quad (9)$$

Similarly, I use (3) and (8) to determine supplier i 's present value:

$$S_i(\rho) = \frac{1}{1-\delta} \int_0^{c(\rho)} [1 - F(c)]^{N-1} F(c)dc \quad (10)$$

Armed with (9) and (10), Proposition 1 leads us to:

Proposition 2. *For $\rho \leq \rho^u$, both $B(\rho)$ and $S_i(\rho)$ are increasing in ρ .*

Proposition 2 implies that both the procurer and the suppliers strictly prefer the project to progress. To better understand suppliers' incentives, we use (8) and re-write the equilibrium payment function previously stated in (2):

$$P_i(\rho, c_i) = \lambda_i(\rho, c_i)c_i + \int_{c_i}^{c(\rho)} \lambda_i(\rho, \tilde{c}_i)d\tilde{c}_i - \delta \sum_{k=1}^N \lambda_k(\rho, c_i)\Delta S_i(\rho) \quad (11)$$

¹⁶Two comments are in order here. First, the buyer would be better off if she could impose a more severe threat of abandoning the project altogether, whenever a supplier refuses to participate, i.e., $R_i(\rho) = 0$ for all ρ . However, given that the procurer can only commit to the current contract, such a threat is not feasible in the present setting. Second, the threat in (8) is suboptimal in the penultimate state, ρ^u . This is because whenever a supplier is awarded the last subproject, the project is completed, and thus the winner exerts a negative externality on others. This means the optimal threat on nonparticipants in this state is to award the project to another supplier with certainty regardless of its cost. While incorporating this threat does not change the qualitative results in what follows, such a strong threat may not be feasible for a government agency either.

According to the r.h.s. of (11), supplier i is reimbursed for his cost if he is awarded the current subproject. In addition, he receives information rents as reflected in the second term. These rents increase with the cutpoint, $c(\rho)$, which in turn increases with the progress of the project. For short sighted suppliers or suppliers that place no weight on future returns, the first two terms summarize the way payments are made. For long sighted suppliers however, the expected future gains or losses introduce the last component of compensation, which is affected by three factors: the discount factor, the expected probability that subproject ρ is assigned to some supplier, $\sum_{k=1}^N \lambda_k(\rho, c_i)$, and the expected change in supplier i 's surplus in state ρ , $\Delta S_i(\rho)$. For instance, in the penultimate state in which the last subproject is auctioned off, we have $\Delta S_i(\rho^u) < 0$.¹⁷ This means each long-sighted supplier would demand further compensation due to the fear of losing the last opportunity to profit. Put differently, suppliers would internalize the negative effect of the progress in the current auction. In the previous stages however, this incentive is reversed. That is, for $\rho < \rho^u$, since $\Delta S_i(\rho) > 0$, the last term in (11) indicates that the procurer taxes away the future increase in supplier i 's surplus in the current payment. In a sense, suppliers invest in the relationship by offering a discount to the procurer. To isolate this investment behavior, note that each supplier's current profit is defined by $\Pi_i(\rho, c_i) = P_i(\rho, c_i) - \lambda_i(\rho, c_i)c_i$:

$$\Pi_i(\rho, c_i) = \begin{cases} \int_{c_i}^{c(\rho)} \lambda_i(\rho, \tilde{c}_i) d\tilde{c}_i - \delta \Delta S_i(\rho), & \text{if } c_i \leq c(\rho) \\ -\delta G(c(\rho); N-1) \Delta S_i(\rho), & \text{if } c_i > c(\rho) \end{cases} \quad (12)$$

There are two important features in (12): First, if a supplier draws a cost below the cutpoint, he expects to win the subproject with some probability. Yet, for costs close to cutpoint, i.e. $c_i \approx c(\rho)$, the winner also expects to incur a loss with certainty, i.e., $\Pi_i(\rho, c_i) < 0$. Second, if a supplier draws a cost greater than the cutpoint, then he knows that he will not be awarded the subproject. However, according to (12), he is willing to pay the amount $\delta G(c(\rho); N-1) \Delta S_i(\rho)$ to the procurer.¹⁸ In fact, it is possible that all suppliers draw costs greater than the cutpoint and pay this amount, even though *ex post* the subproject is not awarded to any supplier. In both cases, it is clear that suppliers have an investment incentive. This is so despite the fact that the procurer cannot write long-

¹⁷Note that $\Delta S_i(\rho^u) = S_i(\rho^u + 1) - S_i(\rho^u) = -S_i(\rho^u)$.

¹⁸Note that for $N = 1$, this payment is 0. This makes sense because the progress of the project depends only on this supplier, and whenever he draws a cost greater than the cutoff, he expects the project to stall with certainty. The buyer then cannot credibly extract a positive payment from the supplier based on a potential progress. This point becomes perhaps more clear in the next subsection.

term contracts or build a reputation to guarantee a fair return to these investments. Two comments are in order here: First, the somewhat counterfactual result that even the losing suppliers make a payment to the procurer can be viewed as an entry fee to the auction (see the subsection below). Second, recall that the reason why a supplier cannot avoid paying the procurer by simply not participating is because doing so will induce the procurer to stall the project. We know however from Proposition 2 that suppliers expect to earn increasingly greater rents along with the progress of the project, as the buyer would then attach more value to it and be more generous. In a sense, suppliers invest in the buyer’s valuation to rip off when it is sufficiently high.

The observation that losing suppliers are willing to make positive payments to the procurer is consistent with Jehiel et al. (1996). In a one-period auction setting, Jehiel et al.(1996) consider a seller who auctions off an object, such as a nuclear weapon, to maximize revenues. However, unlike a standard auction setting, each potential buyer exerts an identity dependent negative externality on nonacquirers. They find that by optimally threatening each buyer to award the project to his “worst” fear, the seller extracts surplus also from agents who do not obtain the object, and that the seller is better off by not selling the object at all while obtaining some payments if externalities are much larger than valuations. Although each stage game in my setting can also be considered as an “auction with externalities”, there are several differences from Jehiel et al.: First, externalities in my dynamic setting are determined endogenously in each stage rather than being fixed. Second, in light of Proposition 2, the winning supplier imposes a positive externality on other suppliers by moving the project forward to a state where the buyer leaves greater rents.

2.1 Implementation by a Vickrey Type Auction

Before proceeding to the comparative static analysis, it is useful to see how the optimal mechanism above can be implemented via a dominant strategy auction of the Vickrey type.¹⁹ In each period, both the procurer and firms observe the current state of the project, ρ . Each firm also learns its private cost of undertaking the current task. Then, the procurer determines the cutoff, $c(\rho) = h^{-1}(\delta\Delta W(\rho))$, and commits to an entry fee $\tilde{E}(\mathbf{c},\rho) = \begin{cases} \delta\Delta S(\rho), & \text{if } \min_i\{c_i\} \leq c(\rho) \\ 0, & \text{if } \min_i\{c_i\} > c(\rho) \end{cases}$, as well as a payment structure $\tilde{P}_i(\mathbf{c},\rho) =$

¹⁹I conjecture that modified versions of other standard auctions such as the first-price auction can implement the optimal mechanism as well.

$$\begin{cases} \min \{c(\rho), \min_{j \neq i} \{c_j\}\}, & \text{if } c_i \leq \min \{c(\rho), \min_{j \neq i} \{c_j\}\} \\ 0, & \text{otherwise} \end{cases}$$
. Unlike the static Vickrey auction, the procurer also needs to discourage nonparticipation in the dynamic setting to correct the incentives for participating bidders. For instance, a bidder whose cost is above the threshold knows that he will not win the current auction. Thus, such a bidder will have a strict incentive not to participate if doing so enables him to avoid the entry fee. If this behavior is not prevented by the procurer, then the bidders whose costs are close to but less than the threshold might also find it optimal not to participate, given that their chances of winning is small. One way of correcting the incentives is for the procurer to commit to stalling the auction for one period unless all bidders participate.

The intuition behind the entry fee is that whenever an admissible cost i.e., below the cutoff, is reported by some firm, the subproject ρ is assigned with certainty. This ensures the progress of the project, and creates an additional surplus to all suppliers. The procurer taxes away this increase in surplus by the entry fee. Conversely, if no admissible cost is reported, no progress will be made in the current period, and thus the fee will be 0. The payment structure is also intuitive: Like in a one-shot Vickrey auction, the winner is paid the second highest cost unless this cost is above the cutoff. It is easy to see that this auction induces truth-telling as a dominant strategy and selects the lowest-cost supplier. Furthermore, it yields the same expected rents to the suppliers and the procurement cost to the buyer as the optimal mechanism.

3 Comparative Statics

In this section, I consider two important comparative statics: First, I determine how the procurer would change her optimal strategy over the course of the project if she faced more suppliers in each stage, and more importantly I determine whether or not the procurer and suppliers would prefer greater competition in each stage. Second, I explore whether the procurer prefers to deal with more patient suppliers who care more about future returns.

3.1 The Effects of The Number of Suppliers

To better see the effects of the number of suppliers in the dynamic setting, first consider a one-period procurement setting. Thus, suppose the buyer auctions off a project that yields a value $V > 0$ to her if it is completed in the current period. Otherwise, the project is abandoned and all parties receive 0 returns. This one-period setting is a special case of the

dynamic setting with $B(\rho) = S_i(\rho) = 0$ for $\rho \leq \rho^u$. Thus, the analysis above remains valid. In particular, the procurer follows the same optimal selection rule in (6) with the cutpoint c_0 satisfying:

$$h(c_0) = \delta V \tag{13}$$

The important observation here is that the cut-off in a one-period setting is independent of the number of suppliers. This is the so-called separation principle in procurement, e.g., Laffont and Tirole (1987), McAfee and McMillan (1987), Riordan and Sappington (1987), suggesting that the winner’s cost is the same as the one that would emerge if the winner faced no competition. Having additional suppliers reduces only the fixed transfer to the winner. When suppliers bid for a large-scale project repeatedly however, an additional supplier might change the incentives for the existing suppliers and for the procurer by changing their expectations about the progress of the project. These expectations affect the endogenous valuation of the project for parties, which in turn affects the current auction design by the procurer. The following result records how the procurer optimally changes the auction design in each stage:

Proposition 3.

- a) $c(\rho^u; N + 1) < c(\rho^u; N)$,
- b) *For sufficiently large ρ^u , there exists ρ_0 such that for $\rho < \rho_0$, $c(\rho; N + 1) > c(\rho; N)$.*

Proposition 3 implies that the separation principle no longer holds in the dynamic setting. The procurer optimally changes the selection rule in (6) in response to an additional supplier. Proposition 3 also implies that the direction of this change depends crucially on the state of the project. In the initial stages, the buyer softens the competition among suppliers by including more “types” in the auction. Although, by doing so, she leaves greater information rents to the winning supplier, she also ensures that the project moves forward in the states where parties do not highly value the project. As the project matures however, future profit opportunities for the existing suppliers are reduced with an additional supplier, and thus they demand greater compensation from the buyer. The buyer mitigates this incentive by reducing the cut off cost thereby increasing competition in those states.

What does this nonmonotonicity imply for parties’ preferences toward greater competition? For instance, would the existing suppliers like to be a monopoly against the procurer and would the procurer like to have as many suppliers as possible throughout the project? The following proposition answers these questions:

Proposition 4.

a-i) $S_i(\rho^u; N + 1) < S_i(\rho^u; N)$,

a-ii) For sufficiently large ρ^u , there exists ρ_0 such that for $\rho < \rho_0$, $S_i(\rho; N + 1) > S_i(\rho; N)$,

b) For $\rho \leq \rho^u$, both $B(\rho; N)$ and $W(\rho; N)$ are increasing in N .

Part (a) of Proposition 4 indicates that firm i 's expected surplus is nonmonotonic in the number of suppliers. In particular, each supplier would favor more suppliers in the initial stages of the project. This is in sharp contrast with the one-period setting in which suppliers are always worse off by the presence of additional suppliers—simply because this lowers the expected transfer from the buyer by reducing the probability of winning. While an additional supplier might reduce the likelihood of winning the current auction in the dynamic setting, it also ensures the progress of the project. As the project moves forward, its value to the buyer increases, leading her to become more willing to make greater payments. However, in order to receive these high payments in mature states, each supplier would then prefer to be a monopoly against the buyer, as recorded in part (a-i).

As alluded to the Introduction, there are two practical implications of part (a-ii). First, for sufficiently long projects, existing firms have an incentive to actively search for new participants to the auction along with the procurer. This is especially interesting in light of Bulow and Klemperer (1996) who argue that attracting one more bidder might be more valuable to the procurer than having more bargaining power. However, in their setting, it is only the auctioneer who has an incentive to search for new bidders. Second, an existing firm might prefer to enter the procurement auction as two or more independent smaller firms.

According to part (b), the procurer always prefers to have more suppliers in each stage of the project. To gain intuition, let us explicitly introduce the parameter N and re-write the procurer's expected surplus as:

$$B(\rho; N) = \frac{1}{1 - \delta} \int_0^{c(\rho; N)} h'(c)G(c; N)dc$$

It is clear that an increase in N introduces two effects: First, it becomes more likely that the winner will draw a lower cost, i.e., $G_N(c; N) > 0$. This is the direct (positive) effect on the procurer's surplus, which would be the only effect in a one-period model. Second, in light of Proposition 3, it leads the buyer to optimally change her selection strategy. In the initial stages, the buyer chooses a higher cut-off in response to a greater N . While this leaves

greater information rents to the winning supplier, it also ensures the progress of the project. In the mature states however, the buyer decreases the cutoff cost with N . Although this might slow down the progress of the project, it also allows the buyer to reduce the transfer to the winning supplier.

I further demonstrate the nonmonotonicity of $c(\rho; N)$ and $S_i(\rho; N)$ within an example as depicted in Figure 2 and 3. In the example, there are 5 stages to complete the project. The project yields a value $V = 3$ to the procurer upon its completion and suppliers' costs are distributed uniformly in $[0, 4]$. Both the procurer and suppliers discount future returns by $\delta = .9$.

As Figure 2 illustrates, one more supplier leads the procurer to increase the cut-off in the initial three stages and reduce it in the remaining two stages. While increasing the cutoff cost softens competition, it also facilitates the progress of the project. For the existing suppliers, an additional supplier has a direct effect of reducing the probability of winning. However, since it also lets the procurer increase the cutoff cost in the initial stages, the overall effect on suppliers' surplus is ambiguous. In fact, as Figure 3 reveals, in the first two stages, each supplier favors one more supplier in the procurement competition. In the third stage, adding a third supplier reduces the surplus below a two supplier competition. In the remaining two stages, since the procurer reduces the cutoff cost and thus increases competition, both the direct and indirect effects move in the same direction and thus lower suppliers' surplus.

3.2 Supplier Discounting and the Project Size

In this section, I ask the following question: would the procurer be better off dealing with more patient suppliers? To answer this question, consider first a one-stage project. Recall that the winning supplier exerts a negative externality on others in the last stage, i.e., $\Delta S_i(\rho^u) < 0$. Since more patient suppliers with a higher discount factor place more weight on this negative change, they would demand higher payments than less patient suppliers.²⁰ Thus, it is intuitive that for a one-stage project, the procurer would prefer suppliers with low discount factors. Consider now a large-scale project. In light of Proposition 2, suppliers' surplus grows as the project moves forward. While the impatient suppliers would place little weight on future surplus, more patient suppliers would internalize this positive future

²⁰This is clearly seen in the payment function given by (11).

effect and thus would be willing to offer lower prices to facilitate the progress of the project. This means the procurer faces a trade-off when dealing with patient suppliers in a long-term project: On the one hand, they offer lower prices in the initial stages of the project, benefitting the procurer. On the other hand, they raise the prices in the mature stages. It seems intuitive that which effect dominates depends on the length of the project. In particular, for a sufficiently large project, the sum of initial price discounts should dominate the future price increases.

To analyze these issues formally, let us assume that all suppliers have a discount factor β while the procurer's discount factor continues to be δ . The following proposition is the main result of this subsection:

Proposition 5. *For $0 \leq \beta_0 < \beta_1 < 1$,*

a) $B(\rho^u; \beta_1) < B(\rho^u; \beta_0)$.

b) For a sufficiently large ρ^u , there exists ρ_0 such that for $\rho < \rho_0$, $B(\rho; \beta_1) > B(\rho; \beta_0)$.

From the procurer's point of view, Proposition 5 highlights the important relationship between suppliers' discount factors and the size of the project. While she would prefer to deal with patient suppliers for sufficiently large projects to exploit their investment incentives early on, she would be better off dealing with less patient suppliers for small-scale projects. Of course, this raises the obvious question: why doesn't the procurer deal with patient suppliers early on and enjoy price discounts and then deal with the less patient suppliers to prevent undue price increases in future stages? While the present analysis does not take the discount factor as a choice variable for the procurer in each stage, it is clear that even a patient supplier who realizes that he is very likely to be replaced in the future would act like a less patient supplier and raise the price. Thus, it is the total expected prices that matter.

To illustrate the result in Proposition 5, I reconsider the example introduced in Section 3.2 with two suppliers. Figure 4 shows the buyer's expected value of the project when suppliers possess discount factors $\beta = .9$ and $\beta = .5$. It is clear that if the project has one or two subprojects, then the buyer is better off dealing with less patient suppliers. However, if the project is larger, then more patient suppliers would yield a higher expected value to the buyer.

4 Benchmark: Complete Information

In the procurement model described above, suppliers are assumed to have private information about their current production costs. This gives them an opportunity to exaggerate their costs to the buyer and secure larger payments as a result. To understand how the buyer exploits competition between suppliers to reduce procurement costs, I now consider a first-best situation where suppliers current costs are common knowledge. The analysis of this case is similar to that above except now no incentive compatibility constraints are imposed on the optimal contract. Formally, the procurer solves the following dynamic program:

$$B^*(\rho) = \max_{\{\lambda_i(\cdot), P_i(\cdot)\}} E_c \sum_{i=1}^N [-P_i^*(\rho, c) + \delta B^*(\rho) + \delta \lambda_i^*(\rho, \mathbf{c}) \Delta B^*(\rho)] \quad (14)$$

subject to $S_i^*(\rho, c_i) \geq \delta R_i^*(\rho)$

Since the buyer's objective requires that she leave the least surplus to the suppliers, she sets the payments so that the constraint binds for all realizations of costs. Inserting these payments into (14) reduces the program to:

$$(1 - \delta)B^*(\rho) = \max_{\{\lambda_i(\cdot)\}} \sum_{i=1}^N E_c \lambda_i^*(\rho, \mathbf{c}) [-c_i + \delta \Delta W^*(\rho)] - \sum_{i=1}^N \delta [R_i^*(\rho) - S_i^*(\rho)] \quad (15)$$

Eq.(15) implies that the optimal strategy for the buyer is given by:

$$\lambda_i^*(\rho, c) = \begin{cases} 1, & \text{if } c_i \leq \min \{c^*(\rho), \min_{j \neq i} \{c_j\}\} \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

where $c^*(\rho) = \delta \Delta W^*(\rho)$.

Comparing (6) and (16), we observe that the procurer selects the lowest cost supplier in each setting. Thus, there is no inefficiency in terms of the price paid to the winning supplier. However, we need to check if the project is assigned at equal frequency. That is, we need to compare the cut off costs. To find the equilibrium in the first-best case, define the following function:

$$\Phi^*(x) = \frac{1 - \delta}{\delta} x + \int_0^x G(c; N) dc \quad (17)$$

Lemma 3.

- a) $\Phi^*(0; \delta) = 0$, $\Phi^{*'}(x; \delta) > 0$ for all $x > 0$, and $\Phi^{*'}(0; \delta) = \frac{1 - \delta}{\delta} \geq 0$.
- b) For $\rho < \rho^u$, $\overline{W}^*(\rho + 1) = \Phi^*(c(\rho); \delta)$, $\overline{W}^*(\rho) = \Phi^*(c(\rho); 1)$ where $\overline{W}^*(\rho) = (1 - \delta)W^*(\rho)$.

Lemma 3 provides the properties of $\Phi^*(x)$ and shows how the backward induction works in the benchmark situation. The following proposition compares the previous competitive bidding case with the complete information benchmark.

Proposition 6. *There exists a unique MPE of the dynamic procurement game with complete information, and it has these properties:*

- a) *Suppliers receive 0 surplus, i.e., $S_i^*(\rho) = 0$.*
- b) *For $\rho \leq \rho^u$, $W(\rho) < W^*(\rho)$ and $B(\rho) < B^*(\rho)$.*
- c) *For $\rho \leq \rho^u$, $c(\rho) < c^*(\rho)$.*

According to Proposition 6, since the procurer can perfectly observe suppliers' costs, she leaves no rents and thus each supplier receives 0 surplus. Not surprisingly, this allows the buyer to generate greater surplus for herself than in the competitive bidding case. However, note that the reason for the inefficiency with the competitive bidding is not because the buyer selects a higher cost supplier, but because she assigns the project too infrequently as reported in part (c) of Proposition 6. This means that the project progresses too slowly in the competitive bidding from a social standpoint.

5 Extensions

In this section, I extend the previous analysis in two ways. First, I analyze the effects of some factors on the optimal division of a project, and second I allow firms to exert cost reducing effort in the tradition of Laffont and Tirole (1993).

5.1 Optimal Division of a Large-Scale Project

In the analysis thus far, the division of the project is taken as given. This seems reasonable when the procurer cannot exceed a predetermined budget in each period and/or faces severe scheduling constraints, for example. However, absent such concerns, dividing a large project into pieces might still benefit the procurer in several ways. First, the procurer can take advantage of repeated competition. Second, suppliers with insufficient resources for the entire project might be enticed to participate. Third, there may be decreasing returns to scale in production.²¹

²¹Papers by Anton and Yao (1987), and Rob (1986) focus on other reasons for why the procurer might stage a large project. Specifically, in their models, the first part of the project is contracted out with a sole-source who improves its technology through learning-by-doing or R&D, and then the second part of the project is auctioned off among multiple firms. The main reason behind staging a project then is that the initial technological improvement is, at least partially, transferrable to the future winner.

To gain intuition and motivate the analysis of the project division, let us re-consider the numerical example introduced above. Suppose a local government plans to contract out a road construction project with value $V = 3$ to $N = 2$ suppliers. The entire project requires 1 unit of input. If the project is auctioned off as one piece, then the winning supplier purchases all the input at the current price, c_i , or at least, the procurer and suppliers can only bargain over this price.²² As before, c_i is uniformly distributed in $[0, 4]$. However, the project can also be divided into 2 equal subprojects, each of which requires, $1/2$ unit of input, and auctioned off in a sequence. Which alternative is better for the procurer? The numerical analysis shows that in the first alternative where the project is auctioned off as one piece, the procurer chooses a cutoff $c(1) = .51$, which means the probability that it will be completed in a given period is $G(c(1); 2) = .24$, and thus the average completion time is $w(1) = \frac{1}{G(c(1); 2)} = 4.17$ periods. Furthermore, the expected payment is $P(1) = .25$, and as a result, the procurer's expected payoff before the auction is $B(1) = 1.26$.

In the second alternative where the project is auctioned off in two pieces, we have the following: For the first subproject, $\rho = 1 : c(1) = .64, G(c(1); 2) = .29, w(1) = 3.41, P(1) = .015$ and $B(1) = .96$. For the second subproject, $\rho = 2 : c(2) = .79, G(c(2); 2) = .36, w(2) = 2.82, P(1) = .06$ and $B(2) = 1.45$. Comparing the two outcomes, the procurer would prefer to auction off the entire project as one piece because it yields an expected payoff of 1.26, which is greater than .96. As will be more clear below, two opposite forces are at work: First, by dividing the project, the procurer takes advantage of the repeated competition as reflected in the lower payments, but second,, the procurer needs to wait longer to complete the project. Overall, the latter negative effect dominates the positive competition effect. How general is this conclusion? To answer this question, I now generalize the example.

Once again, the procurer first decides whether to auction off the project as a whole or divide it into subprojects. If it is divided, then subprojects are procured in a sequence, as is previously analyzed. Like in the example, suppose the project requires 1 unit of input. The unit price of input for supplier i is c_i . The assumptions on c_i are as in Section 2. Suppose that the procurer has the option to divide the project into $k \in \{1, 2, \dots\}$ equal subprojects, each of which requires $\frac{1}{s(k)}$ unit of input, where $s(1) = 1$ and $s(k + 1) \geq s(k)$. Upon dividing the project into pieces, whether or not the required input for each piece decreases

²²If the procurer could contract out the entire project, and bargain up front over possible future changes in input prices, then it is trivial that she would like to auction off the entire project before suppliers gain private information about their future costs. However, such a contract would require that the procurer dictate the amount of input to be purchased regardless of the actual price realization.

proportionally determines the degree of scale economies in the production or service. We say that there are constant (increasing) (decreasing) returns to scale if $s(tk) = (<)(>)ts(k)$ for all $t > 1$. The formal analysis of this case is exactly the same as in Section 2 except that supplier i 's cost for a subproject is $c_{ik} = \frac{c_i}{s(k)}$ rather than being c_i . The following result shows the robustness of the example:

Proposition 7. *Suppose there are constant or increasing returns to scale. Also, suppose the number of firms remains unchanged by the division of the project. Then, the procurer strictly prefers to auction off the entire project without dividing it.*

Proposition 7 reveals that if there are constant or increasing returns to scale in production, then contracting out the entire project at once more than offsets the potential benefits of repeated competition. This is a rather surprising result. Perhaps one would have guessed that the result depend on firms' cost distribution; because whereas dividing the project into pieces might cause a delay in finishing the project, the procurer could nonetheless take advantage of repeated competition. Thus, it is a priori not clear which effect dominates. To see the intuition behind the result and its proof, consider a project of value V and a technology with constant returns. Suppose the project is divided into two subprojects each of which requires $1/2$ unit input. To isolate the two opposing effects of the division, I make the following observation whose general form²³ is proved in the Appendix: From the procurer's perspective, completing a project of value V in two pieces each of which requires $1/2$ unit input is equivalent to completing a project of value $2V$ in two pieces each of which requires 1 unit input. The fact that the final value of the project looks doubled shows the cost savings due to the repeated competition. The procurer now needs to compare two alternatives: Either procure a one piece project with value V or procure a two-piece project with value $2V$, where all pieces require 1 unit input each. Due to the sequential nature of the procurement, the latter alternative is equivalent to procuring the first piece of the project with a continuation value. Since Proposition 1 implies that the total surplus increases at an increasing rate, this continuation value must be strictly less than V for the total surplus and therefore for the buyer's surplus too.²⁴ Thus, procuring the project in two pieces is strictly less preferable to the procurer than procuring it as a whole. The fact that the total

²³The general form of this observation (see the proof of Proposition 7) is that from the buyer's perspective, procuring a project of value V in one piece is equivalent to procuring a project of value $s(k)V$ in k pieces, where each piece requires 1 unit input.

²⁴Mathematically, we essentially have an increasing and strictly convex function whose final value is $2V$. Then it is clear that its value at the mid-point will be strictly less than V .

surplus increases at an increasing rate is an indication of the delay caused by the division of the project.

Perhaps what is equally surprising is that since the result in Proposition 7 relies only on the properties of the total surplus, it is also robust to the way the surplus is shared between the procurer and firms. That is, although I identified the optimal punishment the procurer imposes on the nonparticipating suppliers, the equilibrium total surplus in each stage of the project is independent of this or any other threat. For instance, under the conditions of Proposition 7, the procurer would still prefer to purchase the project in one piece, if she had the commitment power to threaten the nonparticipating suppliers by abandoning the project altogether.

The overall message of Proposition 7 is that for the division of a project to be beneficial to the procurer, there must be either decreasing returns to scale, e.g., capacity constraints or an increase in the number of firms participating in the procurement competition.

Continuing on the numerical example. The procurer would be better off if either the division attracted at least 3 more firms to the competition or there were significant scale diseconomies. To see the latter, suppose $s(k) = k^\alpha$, $\alpha > 1$. In such a case, even though the number of firms remains unchanged, the procurer would benefit from the division for $\alpha \geq 2.2$.

5.2 Laffont-Tirole Extension

In the procurement model described in Section 2, firms' costs come from an exogenous distribution function in each period. That is, firms are assumed to have no discretion over their production costs other than their private knowledge. While this seems a good approximation for cases in which firms rely on outside market for their inputs such as labor and raw material, it is conceivable that they can exert some effort to reduce their baseline costs. This effort can be in the form of searching for a lower input price, allocating the workload more efficiently, or designing a better project plan. To incorporate cost-reducing effort, I extend the model in the tradition of Laffont and Tirole (1993). Specifically, I now envision c_i as firm i 's baseline cost to be obtained without any effort. However, if firm i elects to exert effort, e_i , after being selected, then the resulting cost becomes

$$C(c_i, e_i) = c_i - e_i$$

where $\varphi(e_i)$ denotes the cost of effort with $\varphi', \varphi'' > 0$, $\varphi''' \geq 0$, $\varphi(0) = 0$, $\lim_{e_i \rightarrow c_i} \varphi(e_i) =$

$+\infty$.²⁵

Following Laffont and Tirole, I assume that the procurer can only observe the resulting cost and thus contracts are based on $C(c_i, e_i)$. It turns out that the analysis and results of this model are qualitatively the same as the previous model without effort. Thus, I relegate most of the analysis to the appendix. Here, I provide the procurer's program after taking the (\mathbb{IR}_i) and (\mathbb{IC}_i) constraints into account.

$$(1 - \delta)B(\rho) = \max_{\{\lambda_i(\cdot)\}} E_c \sum_{i=1}^N \lambda_i(\rho, c) [-H(c_i, e(c_i, \rho)) + \delta \Delta W(\rho)] - \sum_{i=1}^N \delta [R_i(\rho) - S_i(\rho)] \quad (18)$$

where $H(c_i, e_i) = c_i - e_i + \varphi(e_i) + \varphi'(e_i) \frac{F(c_i)}{f(c_i)}$ and

$$e(c_i, \rho) = \arg \max_{e_i} H(c_i, e_i). \quad (19)$$

Since $\frac{d}{dc_i} H(c_i, e(c_i, \rho)) > 0$, it is clear that the buyer follows the same optimal strategy in (6) except now the cutoff cost is determined by $H(c(\rho), e(c(\rho), \rho)) = \delta \Delta W(\rho)$. Note also that eq.(19) implies that for a given c_i , firm i exerts the same level of effort ex post regardless of the state if it wins the auction. This is again the basis for the separation principle. The dynamics of the model affect only the number of types included in each state. Since each winning type exerts positive effort in equilibrium, we say that as the cut off increases, so does the winner's effort. Given that the qualitative results of this model regarding the equilibrium sequence of $\{c(\rho)\}$ are the same as in the previous model, I only report the results regarding the effort level:

Proposition 8. *There exists a unique MPE with the Laffont-Tirole extension and it has these properties:*

- a) $e(c_i, \rho)$ increases as the project moves forward,
- b) $e(c_i, \rho^u; N + 1) < e_i(c_i, \rho^u; N)$, and for sufficiently large ρ^u , there exists ρ_0 such that for $\rho < \rho_0$, $e_i(c_i, \rho; N + 1) > e_i(c_i, \rho; N)$.
- c) $e(c_i, \rho) < e^*(c_i, \rho)$ where e^* denotes the first-best effort level.

The intuition behind Proposition 8 comes from the link between the information rents the winning supplier commands and the effort level. Furthermore, since the change in information rents is directly related to the cut-off, and the effort increases as the winning firm earns more information rents, the results follow easily from the previous findings on the equilibrium cut-off costs.

²⁵These assumptions insure that effort is positive for all cost types. However, like in Laffont and Tirole, there is no difficulty or change in the qualitative results once effort is allowed to 0 for some types.

6 Concluding Remarks

This paper presented a rich dynamic procurement model of how a buyer with limited commitment power procures a large-scale project piecewise. While several new insights have emerged from the analysis, the model is simplistic in many ways and open to further extension. For one, as was mentioned in the Introduction, the winner of a subproject may gain experience and be a “strong” bidder in future auctions. Following Lewis and Yildirim (2002a, b), this feature can easily be introduced. Yet, I believe new insights would emerge as to how the procurer would handicap the efficient bidder along with the progress of the project, whether or not the buyer will get locked into one supplier, and how this will affect the optimal division of the project.

Another interesting extension would allow for suppliers with different discounting. This is especially relevant when some suppliers are more likely to stay in business longer than others.²⁶ Based on the findings in Section 3.2, I conjecture that the procurer would set a high entry standard, e.g., a high entry fee, to deal only with suppliers with low discounting when the project is sufficiently large. Otherwise, small firms will also be encouraged to participate.

I believe insights from my model can prove to be useful in other—seemingly unrelated—settings. For instance, one can envision an oligopoly model where firms producing substitute goods or services enter into a new market, e.g., cellular phone. As buyers learn about the new product, say through advertising, they become more willing to pay for the service.²⁷ Since firms offer substitute products, the information buyers gain from using one firm’s product becomes a public good for other suppliers. This is because the informed buyers may not necessarily purchase the service from the previous supplier in the next period. As in the present model, interesting dynamics may arise as the uncertainty about the product is resolved over time. I conjecture that firms would favor the entry of more firms in the initial phases of the market, and would like to be a monopoly in the mature market. The intuition is that one more firm might discourage the existing firms from offering a penetration price via severe discounts due to a reduction in their market share. However, this might benefit the existing firms by allowing them to raise their initial prices in a credible way, and thus

²⁶Jofre-Bonet and Pesendorfer’s (2000) notes this feature in their empirical analysis of California highway construction auctions.

²⁷Rob (1991) considers a model in this direction but with perfect competition. Bergemann and Valimaki (1996) analyze a strategic learning model with two firms only.

alleviating a prisoners' dilemma in giving early discounts. This means more competition initially may not be welfare enhancing.²⁸

7 Appendix A

Proof of Lemma 1. Using the standard techniques, e.g., Fudenberg and Tirole (1991, ch.7), it is easy to see that the second-order condition $\frac{\partial^2 S_i(\rho, c_i)}{\partial c_i^2} \leq 0$ for supplier i 's maximization in (IC_i) requires that $\lambda_i(\rho, c_i)$ be weakly decreasing in c_i . Moreover, since from (1) $S_i(\rho, c_i)$ is weakly decreasing in c_i , (IR_i) is satisfied if and only if $S_i(\rho, \bar{c}) = \delta R_i(\rho)$. Using this boundary condition and integrating (1) over c_i yield

$$S_i(\rho, c_i) = \delta R_i(\rho) + \int_{c_i}^{\bar{c}} \lambda_i(\rho, \tilde{c}_i) d\tilde{c}_i \quad (A1)$$

Combining (A1) and (IC_i) , we derive the payment function in (2). In addition, taking expectation of both sides of (A1) with respect to c_i , we find the expected value stated in (3) for supplier i . ■

Proof of Lemma 2. Differentiating $\Phi(x; \delta)$ with respect to x and using Assumption 1, part (a) easily follows.

To prove part (b), note that supplier i 's expected value can be re-written as:

$$(1 - \delta)S_i(\rho) = \delta[R_i(\rho) - S_i(\rho)] + E_{c_i} \left[\lambda_i(\rho, c_i) \frac{F(c_i)}{f(c_i)} \right] \quad (A2)$$

In equilibrium, summing (5) and (A2) for all i , we obtain

$$\bar{W}(\rho) = \sum_{i=1}^N E_c \lambda_i(\rho, \mathbf{c}) [-c_i + \delta \Delta W(\rho)] \quad (A3)$$

where $\rho \leq \rho^u$ and $\bar{W}(\rho) \equiv (1 - \delta)W(\rho)$.

Using the optimal selection rule in (6) and integrating the r.h.s. of (A3) by parts yield $\bar{W}(\rho) = \Phi(c(\rho); 1)$. Furthermore, $h(c(\rho)) = \delta \Delta W(\rho)$ implies that $\bar{W}(\rho + 1) = \Phi(c(\rho); \delta)$ where $\Phi(\cdot)$ is as defined in (7). ■

Proof of Proposition 1. I use backward induction. Since $\Phi(0; \delta) = 0$, $\Phi(\bar{c}; \delta) > (1 - \delta)V$ by Assumption 2, and $\Phi'(x; \delta) > 0$ for $x > 0$, there exists a unique $c(\rho^u) \in (0, \bar{c})$ that solves $\Phi(c(\rho^u); \delta) = \bar{W}(\rho^u + 1) = (1 - \delta)V$. Now consider $\rho = \rho^u - 1$. By using

²⁸Compte and Jehiel (2002) consider an interesting one-period auction model with affiliated values in which more competition might mean lower welfare.

similar arguments, one can conclude that there exists a unique $c(\rho^u - 1) \in (0, \bar{c})$ that solves $\Phi(c(\rho^u - 1); \delta) = \overline{W}(\rho^u) = \Phi(c(\rho^u); 1) < (1 - \delta)V$. Furthermore, since $\Phi(c(\rho^u); 1) < \Phi(c(\rho^u); \delta)$, we have $c(\rho^u - 1) < c(\rho^u)$. To complete the induction argument, suppose, for some ρ , there exists a unique $c(\rho) \in (0, \bar{c})$. The exact arguments above imply the existence of a unique $c(\rho - 1) \in (0, \bar{c})$ that solves $\Phi(c(\rho - 1); \delta) = \overline{W}(\rho)$. This completes the proof of part (a).

To prove part (b), recall that $h(c(\rho)) = \delta\Delta W(\rho)$. Since $c(\rho) > 0$ and $h'(\cdot) > 0$, the result follows. ■

Proof of Proposition 2. Follows directly from part (a) of Proposition 1, and equations (9) and (10). ■

Before proceeding to the proofs of Proposition 3 and 4, we first record the following useful result.

Lemma A1. *For sufficiently large ρ^u , there exists ρ_0 such that for $\rho < \rho_0$, $\lim_{\rho^u \rightarrow \infty} c(\rho) = 0$.*

Proof. Note that the sequence $\{c(\rho)\}_{\rho=1}^{\rho=\rho^u}$ is an increasing or $\{c(\rho)\}_{\rho=\rho^u}^{\rho=1}$ is a decreasing sequence. Furthermore, since the latter is bounded below by 0, it converges to some $\alpha \geq 0$. Since every subsequence of a convergent sequence also converges to the same limit, we must have $\lim_{\rho^u \rightarrow \infty} c(\rho) = \lim_{\rho^u \rightarrow \infty} c(\rho + 1) = \alpha$. From the recursion described in Lemma 1 and by continuity of $\Phi(\cdot)$, this implies that $\Phi(\alpha; \delta) = \Phi(\alpha; 1)$, or $\frac{1-\delta}{\delta}h(\alpha) = 0$, whose unique solution is, $\alpha = 0$. ■

Proofs of Proposition 3 and 4. I first show that for $\rho \leq \rho^u$, $W(\rho; N) < W(\rho; N + 1)$. I proceed by induction. Suppose $W(\rho^u; N) \geq W(\rho^u; N + 1)$. Then, since $W(\rho^u + 1; N) = W(\rho^u + 1; N + 1) = V$, we have $\Delta W(\rho^u; N) \leq \Delta W(\rho^u; N + 1)$. This implies $c(\rho^u; N) \leq c(\rho^u; N + 1)$ due to $h(c(\rho)) = \delta\Delta W(\rho)$. However, since $\Phi(x; \delta)$ is increasing in both x and N , we must have $W(\rho^u; N) < W(\rho^u; N + 1)$, which contradicts our original supposition. Thus, $W(\rho^u; N) < W(\rho^u; N + 1)$.

Now suppose $W(\rho; N) < W(\rho; N + 1)$ for some $\rho \leq \rho^u$. By way of contradiction however, suppose also $W(\rho - 1; N) \geq W(\rho - 1; N + 1)$. Since, from Proposition 1, $W(\rho; N)$ is increasing in ρ , we have

$$W(\rho - 1; N + 1) \leq W(\rho - 1; N) < W(\rho; N) < W(\rho; N + 1)$$

which, in turn, implies $\Delta W(\rho - 1; N + 1) > \Delta W(\rho - 1; N + 1)$. From $h(c(\rho)) = \delta\Delta W(\rho)$,

this yields $c(\rho - 1; N + 1) > c(\rho - 1; N)$. However, this means $W(\rho - 1; N) < W(\rho - 1; N + 1)$, a contradiction. Thus, $W(\rho - 1; N) < W(\rho - 1; N + 1)$.

Next, I prove Proposition 3.

Once again, the fact that $W(\rho^u + 1; N) = W(\rho^u + 1; N + 1) = V$ and $W(\rho^u; N) < W(\rho^u; N + 1)$ implies that $\Delta W(\rho^u; N + 1) < \Delta W(\rho^u; N)$, which, together with $h(c(\rho)) = \delta \Delta W(\rho)$, yields $c(\rho^u; N + 1) < c(\rho^u; N)$.

To prove part (b), first note that for x close to 0, the first-order Taylor expansion on $\Phi(x; \delta)$ yields

$$\begin{aligned}\Phi(x; \delta) &\approx \Phi(0; \delta) + \Phi'(0; \delta)x \\ &= \frac{1 - \delta}{\delta} h'(0)x\end{aligned}\tag{A4}$$

From Lemma A1, (A4) reveals that for sufficiently large ρ^u , there exists ρ_0 such that for $\rho < \rho_0$:

$$\overline{W}(\rho; N + 1) \approx \frac{1 - \delta}{\delta} h'(0)c(\rho; N + 1)\tag{A5}$$

$$\overline{W}(\rho; N) \approx \frac{1 - \delta}{\delta} h'(0)c(\rho; N)\tag{A6}$$

Given that $W(\rho; N)$ is strictly increasing in N , (A5) and (A6) imply that $c(\rho; N + 1) > c(\rho; N)$. This completes the proof of Proposition 3.

Now, I complete the proof of Proposition 4.

Recall each supplier's surplus in (10):

$$S_i(\rho; N) = \frac{1}{1 - \delta} \int_0^{c(\rho; N)} [1 - F(c)]^{N-1} F(c) dc\tag{A7}$$

Part (a) of Proposition 3 and (A7) imply that $S_i(\rho^u; N + 1) < S_i(\rho^u; N)$. To prove part (a-ii), define the function: $H(x) \equiv \int_0^x [1 - F(c)]^{N-1} F(c) dc$. For x close to 0, we have

$$\begin{aligned}H(x) &\approx H(0) + H'(0)x + H''(0)\frac{x^2}{2} \\ &= f(0)\frac{x^2}{2}\end{aligned}\tag{A8}$$

by a second-order Taylor expansion. Using Lemma A1 and (A8), we can approximate $S_i(\rho; N)$ for sufficiently small ρ by:

$$S_i(\rho; N) \approx \frac{1}{1 - \delta} f(0) \frac{c^2(\rho; N)}{2}\tag{A9}$$

From (A9) and part (b) of Proposition 3, we obtain the desired result in part (a-ii).

To show that $B(\rho; N)$ is increasing in N . Recall that $B(\rho; N) = \frac{1}{1-\delta} \int_0^{c(\rho; N)} h'(c)G(c; N)dc$. Note that since $c(\rho; \cdot)$ is a choice variable for the procurer, whenever there is one more firm, the procurer could simply choose $c(\rho; N+1) = c(\rho; N)$ and be better off. Since she chooses $c(\rho; N+1) \neq c(\rho; N)$, it must be that $B(\rho; N+1) > B(\rho; N)$. ■

Proof of Proposition 5. Let δ and β be the buyer's and suppliers' discount factors, respectively. Proceeding as in the previous case with the same discount factor, we see that the equilibrium sequence of $\{c(\rho)\}$ is determined by the following recursive equations:

$$\begin{aligned} B(\rho^u + 1) &= V \text{ and } S_i(\rho^u + 1) = 0 \\ B(\rho) &= \frac{1}{1-\delta} \int_0^{c(\rho)} h'(c)G(c; N)dc \\ S_i(\rho) &= \frac{1}{1-\beta} \int_0^{c(\rho)} [1-F(c)]^{N-1}F(c)dc \\ h(c(\rho)) &= \Delta \widetilde{W}(\rho) \end{aligned} \tag{A10}$$

where $\widetilde{W}(\rho) \equiv \delta B(\rho) + \beta \sum_{i=1}^N S_i(\rho)$.

Following the same steps in the proof of Proposition 1, one can show from (A10) that there exists a unique MPE in this case as well. Moreover the previous results in Proposition 1-4 hold. Now to prove the result in Proposition 5, I first show that $\widetilde{W}(\rho; \beta)$ is increasing in β . Take arbitrary β_0 and β_1 such that $\beta_0 < \beta_1$. Now suppose $\widetilde{W}(\rho^u; \beta_1) \leq \widetilde{W}(\rho^u; \beta_0)$. Since $\widetilde{W}(\rho^u + 1; \beta_1) = \widetilde{W}(\rho^u + 1; \beta_0) = \delta V$, (A10) implies that $c(\rho^u; \beta_0) \leq c(\rho^u; \beta_1)$, which in turn implies that $\widetilde{W}(\rho^u; \beta_1) > \widetilde{W}(\rho^u; \beta_0)$. This yields a contradiction. Thus, $\widetilde{W}(\rho^u; \beta_1) > \widetilde{W}(\rho^u; \beta_0)$. To complete the induction argument, suppose for some $\rho \leq \rho^u$ that $\widetilde{W}(\rho; \beta_1) > \widetilde{W}(\rho; \beta_0)$, but on the contrary that $\widetilde{W}(\rho - 1; \beta_1) \leq \widetilde{W}(\rho - 1; \beta_0)$. Since $\widetilde{W}(\rho)$ is increasing in ρ , we have $\widetilde{W}(\rho - 1; \beta_1) \leq \widetilde{W}(\rho - 1; \beta_0) < \widetilde{W}(\rho; \beta_0) < \widetilde{W}(\rho; \beta_1)$. This means $c(\rho - 1; \beta_0) < c(\rho - 1; \beta_1)$. However, (A10) implies that $\widetilde{W}(\rho - 1; \beta_1) > \widetilde{W}(\rho - 1; \beta_0)$, a contradiction. Thus, $\widetilde{W}(\rho - 1; \beta_1) > \widetilde{W}(\rho - 1; \beta_0)$.

To prove part (i) of Proposition 5, note that since $\widetilde{W}(\rho^u; \beta)$ is increasing in β , we have $c(\rho^u; \beta_1) < c(\rho^u; \beta_0)$. Thus, $B(\rho^u; \beta_1) < B(\rho^u; \beta_0)$. To prove part (ii), note that Lemma A1 holds here as well. Thus, a second order Taylor expansion on $\widetilde{W}(\rho; \beta)$ for sufficiently small ρ yields:

$$\widetilde{W}(\rho; \beta) \approx Nf(0) \left[\frac{\delta}{1-\delta} h'(0) + \frac{\beta}{1-\beta} \right] \frac{c^2(\rho; \beta)}{2}$$

Since $\widetilde{W}(\rho; \beta)$ is increasing in β , we have $c(\rho; \beta_0) > c(\rho; \beta_1)$, which implies that $B(\rho; \beta_1) > B(\rho; \beta_0)$. ■

Proof of Lemma 3. Similar to the proof of Lemma 2. ■

Proof of Proposition 6. Since, at the optimal solution, we have $S_i^*(\rho, c_i) = \delta R_i^*(\rho) = \delta S_i^*(\rho)$, taking expectation with respect to c_i reveals that $S_i^*(\rho) = \delta S_i^*(\rho)$ and thus $S_i^*(\rho) = 0$.

Now note that in the incomplete information setting, if both the procurer and the suppliers were maximizing the total surplus, then they would agree to assign the project to the lowest cost supplier in each period if this cost is not too high. This means they would solve the following problem:

$$W(\rho) = \max\{-c_m + \delta W(\rho + 1), \delta W(\rho)\} \quad (\text{A11})$$

where $c_m = \min_i\{c_i\}$.

Given that $S_i^*(\rho) = 0$ in the complete information setting, the total surplus in (A11) coincides with the one in (15). However, since the procurer does not choose the same cutoff as in (A11) when maximizing her own surplus in the incomplete information case, the total surplus generated is strictly less than that of the complete information.

To prove part (c), first observe that $\Phi(x) > \Phi^*(x)$ for all $x > 0$. Since $W(\rho) < W^*(\rho)$, this implies that $c(\rho) < c^*(\rho)$. ■

8 Appendix B

Before proceeding to the proof of Proposition 7, we note the following useful result:

Lemma B1. *Consider the model in Section 2. Suppose two projects, X and Y , are of equal length but with different final payoffs, V_X and V_Y where $V_X < V_Y$. Then, for any $\rho \leq \rho^u$, we have $B_X(\rho) < B_Y(\rho)$.*

Proof of Lemma B1. Given (9), it suffices to show that for any $\rho \leq \rho^u$, $c_X(\rho) < c_Y(\rho)$. Suppose, on the contrary, that $c_X(\rho^u) \geq c_Y(\rho^u)$. This means $W_X(\rho^u) \geq W_Y(\rho^u)$. Since $V_X < V_Y$, we have $c_X(\rho^u) < c_Y(\rho^u)$, a contradiction. Now suppose for some $\rho \leq \rho^u$, $c_X(\rho) < c_Y(\rho)$, but, on the contrary, $c_X(\rho - 1) \geq c_Y(\rho - 1)$. This implies that $W_Y(\rho - 1) \leq W_X(\rho - 1) < W_X(\rho) < W_Y(\rho)$, where the second inequality follows from Proposition 1. However, this implies $c_X(\rho - 1) < c_Y(\rho - 1)$, a contradiction. ■

Proof of Proposition 7. Note first that the distribution and density functions of $c_{ik} = \frac{c_i}{s(k)}$ are given by $F_k(c_{ik}) = F(s(k)c_{ik})$ and $f_k(c_{ik}) = f(s(k)c_{ik})s(k)$, respectively. Second, to find the sequence of $\{c_k(\rho)\}$, we compute $\Phi_k(x_k; \delta)$ by substituting x with x_k and the distribution with $F_k(c_{ik})$ in (7) as:

$$\begin{aligned}\Phi_k(x_k; \delta) &= \frac{1-\delta}{\delta}h_k(x_k) + \frac{F_k(x_k)}{f_k(x_k)}G_k(x_k; N) + \int_0^{x_k} G_k(c; N)dc \\ &= \frac{1}{s(k)}\Phi(x; \delta)\end{aligned}\tag{B1}$$

where we make a change of variable by $x_k \equiv \frac{x}{s(k)}$.

Thus, the following set of equations generates the unique sequence of $\{c(\rho)\}$ where $c_k(\rho) \equiv \frac{c(\rho)}{s(k)}$:

$$W_k(\rho^u + 1) = V; (1-\delta)W_k(\rho + 1) = \frac{1}{s(k)}\Phi(c(\rho); \delta), \text{ and } (1-\delta)W_k(\rho) = \frac{1}{s(k)}\Phi(c(\rho); 1)\tag{B2}$$

Letting $\widetilde{W}_k(\rho) \equiv s(k)W_k(\rho)$, and $\widetilde{B}_k(\rho) \equiv s(k)B_k(\rho)$, we can re-write (B2) as:

$$\widetilde{W}_k(\rho^u + 1) = s(k)V; (1-\delta)\widetilde{W}_k(\rho + 1) = \Phi(c(\rho); \delta), \text{ and } (1-\delta)\widetilde{W}_k(\rho) = \Phi(c(\rho); 1)\tag{B3}$$

Note that (B3) essentially converts a project division problem into the one analyzed in Section 2 by summarizing the effect of division only on the final value. Now we are ready to prove the conclusion of Proposition 7.

Suppose that $s(\rho k) \leq \rho s(k)$ for $\rho > 1$ and that the number of suppliers is unaffected by the division of the project. Take any $k \in \{1, 2, \dots\}$. We want to show that the buyer prefers dividing the project in k subprojects to $k+1$ subprojects, i.e., $B_{k+1}(\rho^u + 1 - (k+1)) < B_k(\rho^u + 1 - k)$. According to (B3), $\widetilde{B}_{k+1}(\rho^u + 1 - (k+1))$ is buyer's expected surplus from a project with $(k+1)$ subprojects and whose final value is $s(k+1)V$. Since the buyer decides optimally at every stage of the project, $\widetilde{B}_{k+1}(\rho^u + 1 - (k+1))$ is also the expected surplus from a project with k subprojects and whose final value is $\widetilde{B}_{k+1}(\rho^u)$. With this interpretation in mind, if we can compare $\widetilde{B}_{k+1}(\rho^u)$ and $s(k)V$, then we reach the conclusion by Lemma B1. Now, recall that from Proposition 1, the total surplus increases at an increasing rate as the project moves forward. Thus, $\widetilde{W}_{k+1}(\rho^u) < \frac{k}{k+1}s(k+1)V$. Letting $\rho = \frac{k+1}{k}$, $s(\rho k) \leq \rho s(k)$ implies that $\frac{k}{k+1}s(k+1) \leq s(k)$. This means $\widetilde{B}_{k+1}(\rho^u) < \widetilde{W}_{k+1}(\rho^u) < s(k)V$. Lemma B1, then, implies that $\widetilde{B}_{k+1}(\rho^u + 1 - (k+1)) < \widetilde{B}_k(\rho^u + 1 - k)$, which in turn implies

$s(k+1)B_{k+1}(\rho^u + 1 - (k+1)) < s(k)B_k(\rho^u + 1 - k)$. Since $s(k+1) \geq s(k)$, we have $B_{k+1}(\rho^u + 1 - (k+1)) < B_k(\rho^u + 1 - k)$. ■

LAFFONT-TIROLE EXTENSION:

Here, I provide the analysis of the Laffont-Tirole extension without detailing the full argument, as the formal analysis follows the same steps as in the preceding case without effort. firm i 's incentive compatibility constraint is now stated as:

$$S_i(\rho, c_i) = \max_{\hat{c}_i} S_i(\rho, \hat{c}_i) \equiv P_i(\rho, \hat{c}_i) - \lambda_i(\rho, \hat{c}_i) [C_i(\hat{c}_i; c_i, \rho) + \varphi(e(\hat{c}_i; c_i, \rho))] \quad (\text{B4})$$

$$+ \delta S_i(\rho) + \delta \sum_{k=1}^N \lambda_k(\rho, \hat{c}_i) \Delta S_i(\rho)$$

where $e(c_i; \rho) \equiv e(c_i; c_i, \rho)$ and $e(\hat{c}_i; c_i, \rho) = c_i - \hat{c}_i + e(\hat{c}_i; \rho)$.

From the Envelope Theorem, this implies²⁹

$$\frac{dS_i(\rho, c_i)}{dc_i} = -\lambda_i(\rho, c_i) \varphi'(e(c_i, \rho)) (\leq 0) \quad (\text{B5})$$

Eq. (B5) and (IR _{i}) reveal that

$$S_i(\rho, c_i) = \delta R_i(\rho) + \int_{c_i}^{\bar{c}} \lambda_i(\rho, \tilde{c}_i) \varphi'(e(\tilde{c}_i, \rho)) d\tilde{c}_i \quad (\text{B6})$$

Combining (B4) and (B6) yield the payment to firm i :

$$P_i(\rho, c_i) = \delta [R_i(\rho) - S_i(\rho)] + \lambda_i(\rho, c_i) [C_i(c_i, \rho) + \varphi(e(c_i, \rho))] \quad (\text{B7})$$

$$+ \int_{c_i}^{c(\rho)} \lambda_i(\rho, \tilde{c}_i) \varphi'(e(\tilde{c}_i, \rho)) d\tilde{c}_i - \delta \sum_{k=1}^N \lambda_k(\rho, c_i) \Delta S_i(\rho)$$

The buyer solves the same problem as in (4) except now (IC _{i}) is as stated in (B4). Inserting the payments from (B7) into the buyer's problem reduces it to (18). Now, taking expectation of both sides of (B6) reveals that $(1 - \delta)S_i(\rho) = \delta [R_i(\rho) - S_i(\rho)] + E_{c_i} \left[\lambda_i(\rho, c_i) \varphi'(e(c_i, \rho)) \frac{F(c_i)}{f(c_i)} \right]$. Define

$$\Psi(x; \delta) = \frac{1 - \delta}{\delta} H(x) + \varphi'(e(x)) \frac{F(x)}{f(x)} G(x; N) + \int_0^x \{1 - [1 - \varphi'(e(c, \rho))] e_c(c, \rho)\} G(c; N) dc \quad (\text{B8})$$

²⁹Having $\frac{d\lambda_i(T, c_i)}{dc_i} \leq 0$ and $e_c(c_i, T) \leq 1$ together constitute a set of sufficient conditions for (IC _{i}) in (B5). It is easy to see that these conditions are satisfied in equilibrium.

where $H(x) = x - e(x) + \varphi(e(x)) + \varphi'(e(x))\frac{F(x)}{f(x)}$.

Note that Ψ possesses the same properties as Φ given in Lemma 2 in the text.³⁰ Furthermore, to find $\{c(\rho)\}$, we only need replace Φ with Ψ . Thus, following similar lines, Proposition 1 and 2 immediately follow for the Laffont-Tirole extension. Since, for $x \approx 0$, a first-order Taylor expansion reveals that $\Psi(x; \delta) \approx \frac{1-\delta}{\delta}H'(0) = \frac{1-\delta}{\delta}[1 + \varphi'(0)\frac{d}{dc}\left[\frac{F(c)}{f(c)}\right]]$, which is independent of N , Proposition 3 and 4 hold as well. Similar arguments also show that Proposition 5 also remains valid. ■

³⁰Note from (19) that $\varphi'(e) < 1$ at the optimal effort.

References

- [1] Anton, J. and D. Yao, 1987, "Second Sourcing and the Experience Curve: Price Competition in Defense Procurement," *RAND Journal of Economics*, vol. 18, 57-76.
- [2] Ashenfelter, O., 1989, "How Auctions Work for Wine and Art," *Journal of Economic Perspectives*, vol. 3, 23-36.
- [3] Bernhardt, D. and D. Scoones, 1994, "A Note on Sequential Auctions," *American Economic Review*, vol. 84, 653-7.
- [4] Bulow, J. and P. Klemperer, 1996, "Auctions versus Negotiations," *American Economic Review*, vol. 86, 180-94.
- [5] Caillaud, B. and C. Mezzetti, 2003, "Equilibrium Reserve Prices in Sequential Ascending Auctions," working paper.
- [6] Compte, O. and P. Jehiel, 2002, "On the Value of Competition in Procurement Auctions," *Econometrica*, vol. 70, 343-55.
- [7] Cremer, J. and R. McLean, 1988, "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*, 56, 1247-58.
- [8] Fudenberg, D. and J. Tirole. *Game Theory*. 1991. MIT Press.
- [9] Gale, I. and D.B. Hausch, 1994, "Bottom-Fishing and Declining Prices in Sequential Auctions," *Games and Economic Behavior*, vol. 7, 318-31.
- [10] Gale, I., D.B. Hausch and M. Stegeman, 2000. "Sequential Procurement with Subcontracting," *International Economic Review*, vol. 41, 989-1020.
- [11] Hoekman, B. M. and P. C. Mavroidis, *Law and Policy in Public Purchasing*, Eds. 1997, Ann Arbor: University of Michigan Press.
- [12] Jehiel, P., B. Moldovanu, and E. Stacchetti, 1996, "How (Not) to Sell Nuclear Weapons," *American Economic Review*, vol. 86, 814-29.
- [13] Jofre-Bonet, M. and M. Pesendorfer, 2000, "Bidding Behavior in a Repeated Procurement Auction: A Summary," *European Economic Review*, vol. 44, 1006-20.

- [14] Krishna, K., 1993, "Auctions with Endogenous Valuations: The Persistence of Monopoly Revisited," *American Economic Review*, vol. 83, 147-60.
- [15] Laffont, J.J. and J. Tirole, 1987, "Auctioning Incentive Contracts," *Journal of Political Economy*, vol. 95, 921-37.
- [16] Laffont, J.J. and J. Tirole, 1988, "Repeated Auctions of Incentive Contracts, Investment, and Bidding Parity with an Application to Takeovers," *RAND Journal of Economics*, vol. 19, 516-37.
- [17] Laffont, J.J. and J. Tirole. *A Theory of Incentives in Procurement and Regulation*. 1993. MIT Press.
- [18] Lewis, T., 1983, "Preemption, Divestiture, and Forward Contracting in a Market Dominated by a Single Firm," *American Economic Review*, vol. 73, 1092-101.
- [19] Lewis, T., 1986, "Reputation and Contractual Performance in Long-Term Projects," *RAND Journal of Economics*, vol.17, 141-57.
- [20] Lewis, T. and H. Yildirim, 2002a, "Learning by Doing and Dynamic Regulation," *RAND Journal of Economics*, vol.33, 22-36.
- [21] Lewis, T. and H. Yildirim, 2002b, "Managing Dynamic Competition," *American Economic Review*, vol. 92, 779-97.
- [22] Luton, R. and P. McAfee, 1986, "Sequential Procurement Auctions," *Journal of Public Economics*, vol. 31, 181-95.
- [23] Manelli, A. and D. Vincent, 1995), "Optimal Procurement Mechanisms," *Econometrica*, vol. 63, 591-620.
- [24] McAfee, R. P. and J. McMillan, 1987, "Competition for Agency Contracts," *RAND Journal of Economics*, vol 18, 296-307.
- [25] McAfee, R. P. and J. McMillan, *Incentives in Government Contracting*. Toronto: University of Toronto Press, 1988.
- [26] McAfee, R. P. and D. Vincent, 1993, "The Declining Price Anomaly," *Journal of Economic Theory*, vol. 60, 191-211.

- [27] McAfee, R. P. and D. Vincent, 1997, "Sequentially Optimal Auctions," *Games and Economic Behavior*, vol. 18, 246-76.
- [28] Myerson, R., 1979, "Incentive Compatibility and the Bargaining Problem," *Econometrica*, vol. 47, 61-73.
- [29] Riordan, M. and D. Sappington, 1987, "Awarding Monopoly Franchises," *American Economic Review*, vol. 77, 375-87.
- [30] Rob, R., 1986, "The Design of Procurement Contracts," *American Economic Review*, vol. 76, 378-89.
- [31] Rob, R., 1991, "Learning and Capacity Expansion under Demand Uncertainty," *Review of Economic Studies*, vol. 58, 655-75.

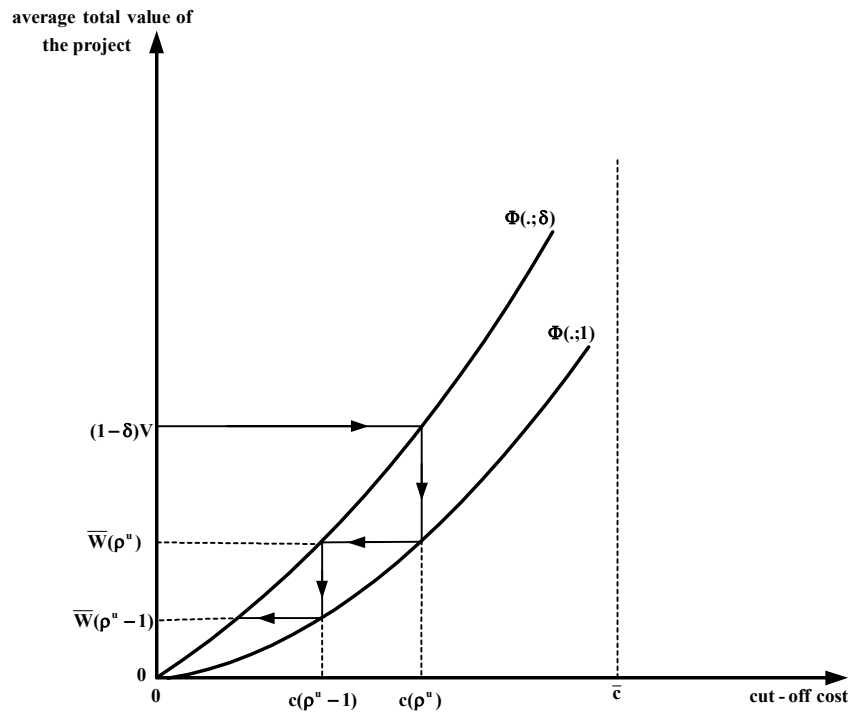


Figure 1

Figure 2

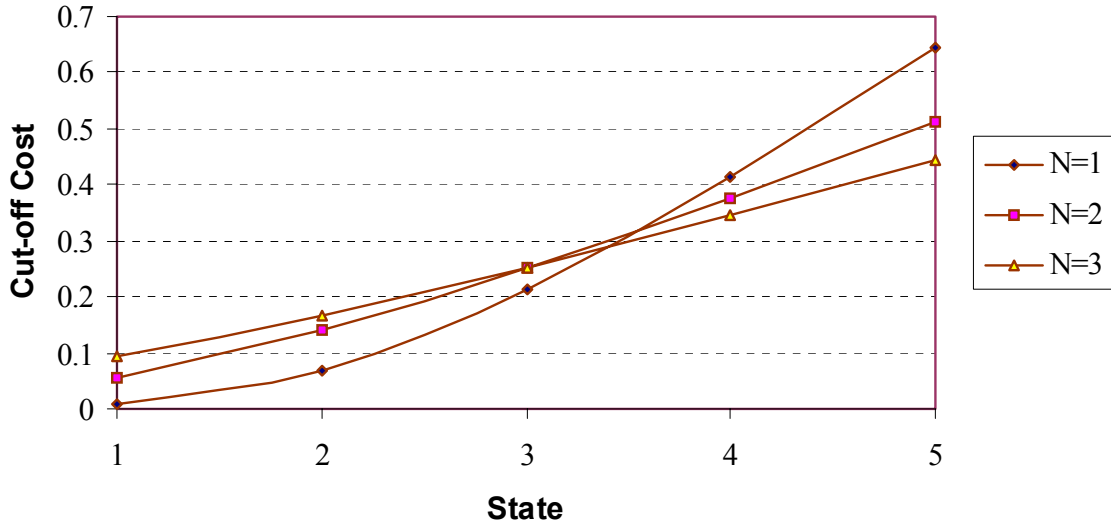


Figure 3

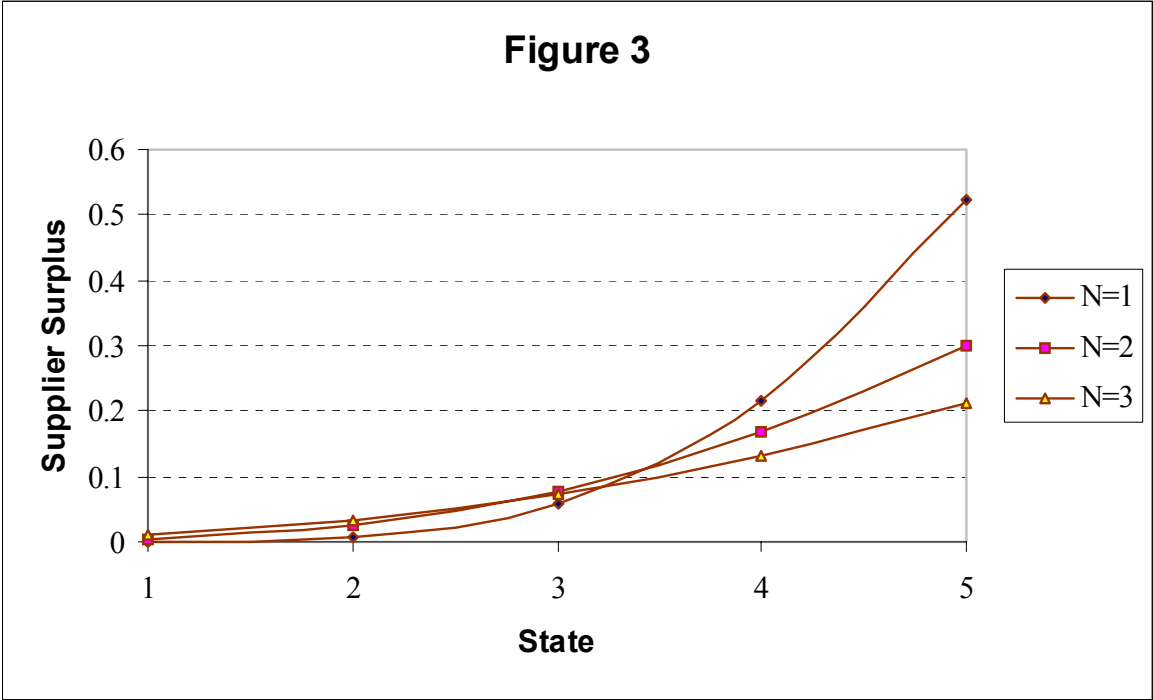


Figure 4

