Contests with Multiple Rounds*

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April 6, 2004

*I thank two anonymous referees, the editors of this journal, Omer Gokcekus, Wolfgang Kohler and Claudio Mezzetti for helpful comments and suggestions. All errors are mine.

Abstract

This paper studies contests where players have the flexibility to add to their previous efforts after observing their rivals' most recent effort in an intermediate stage. It is found that (1) contrary to previous findings, the Stackelberg outcome where the underdog leads and the favorite follows cannot be an equilibrium. (2) There are multiple subgame perfect equilibria all occuring on the underdog's usual one-shot reaction function in-between and including the one-shot Cournot-Nash and Stackelberg outcome with the favorite leading. (3) The total equilibrium effort is typically greater than or equal to what a one-shot Cournot-Nash play would predict; and (4) in settings where players can choose whether or not to disclose their early actions to the rival, both the favorite and the underdog disclose in equilibrium. Applications in sports, lobbying, and R&D races are discussed.

JEL Classification: C72;D72

KEY WORDS: Contests; Cournot-Nash Game; Favorite; Multiple Rounds; Underdog; Stackelberg Game

1 Introduction

Many economic and social settings can be characterized as contests in which agents expend resources or effort to win a given prize. The standard examples include interest groups' lobbying to pass a legislation in their favor, political candidates' competing to claim an office, firms' expending R&D resources to secure a patent or to win a research tournament, senior employees' exerting effort to become a CEO, and athletes' competing to win a grand prize. While the early literature on contests has predominantly studied the one-shot Cournot-Nash model where players choose their effort levels only once and simultaneously, there is a growing strand of this literature recognizing the sequential nature of many contests where one player moves before the other.[1] In an insightful paper, Dixit (1987) compares the effort levels of the Cournot-Nash play with those of the Stackelberg plays. By calling the player who is more likely to win in a one-shot Cournot-Nash game the *favorite,* and the rival the *underdog*, Dixit finds that given the (exogenous) chance to move first, the favorite overcommits to his effort with respect to his Cournot-Nash amount, while the underdog undercommits. This result is important in two respects: First, it demonstrates how unevenly matched players significantly differ in their strategies when they can precommit to their efforts. Second, while the total effort of a Stackelberg game with the favorite's leadership is typically greater than that of the Cournot-Nash play, the opposite holds with the underdog's leadership. Thus, whether the one-shot Cournot-Nash model over- or under-estimates the social cost associated with the rent-seeking depends crucially on which party moves first in a sequential contest. Subsequent papers by Baik and Shogren (1992) and Leininger (1993) have endogenized the order of moves by introducing an announcement stage in which players simultaneously decide and commit to their period of action; and once the timing becomes common knowledge, they either play a Cournot-Nash or one of two Stackelberg games. Surprisingly, they find that the Stackelberg play where the underdog leads is the unique equilibrium. Hence, they conclude that the Cournot-Nash assumption is never realized, and furthermore the Cournot-Nash model overestimates the social cost. These interesting results however rest critically on the assumption that agents can take action only once. While this is a good approximation in a variety of contests with either a large fixed cost of exerting effort or a one-time opportunity of action granted by the institutional design, there are many other contests in which players can exert effort multiple times without pre-announcing. Examples abound. Interest groups in the U.S. can make multiple nonrefundable contributions to politicians, which become public records soon after they are made, giving each group an opportunity to observe the rival's most recent contribution.[2] It is commonly observed that presidential candidates visit the same state multiple times to increase their voter turnout before the campaigning is over. Conceivably, at least some of these visits will be in response to the poll results, which can be a (noisy) performance measure for each candidate relative to the rival's.[3] Similarly, firms participating in a research tournament can increase their chances of discovery by visibly hiring the most talented scientists and multiplying the number of labs.[4] Clearly, how aggressively a firm invests over time will depend on its rival's investment. Finally, athletes competing in multiple categories, e.g., triathlon, decathlon, or all-around, and teams in NBA finals learn their scores after each round and see where they stand with respect to the rival(s), giving them an opportunity to adjust their future efforts.

The objective of this paper is to extend the analysis by Dixit (1987) and capture the three main elements present in all these examples. First, players can exert effort in multiple periods before the contest concludes. Second, they do so by observing their rival's recent effort in an intermediate stage; and third, the probability of winning depends on the cumulative effort levels. By focusing on the subgame perfect equilibria, I find that in an unevenly matched contest, contrary to the previous findings by Baik and Shogren (1992) and Leininger (1993),[5] the Stackelberg outcome in which the underdog leads can never be an equilibrium. This is because playing low ball early on, the underdog would have a strict incentive to increase his effort along with the favorite. Anticipating this behavior by the underdog, the favorite takes action in the first period, thereby curbing the underdog's incentive. This does not mean however that the only equilibrium is the Stackelberg play in which the favorite leads and the underdog follows. There are multiple equilibrium outcomes, all of which are on the underdog's reaction function in-between and including the one-shot Cournot-Nash and the Stackelberg point with the favorite's leadership. Several empirically relevant observations emerge from these equilibria. First, total effort is typically greater than or equal to that of the one-shot Cournot-Nash model, suggesting that the social cost is in general underestimated by this model. Second, while the feasibility of multiple actions benefits the favorite, it hurts the underdog. Third, in all equilibria except for the Cournot-Nash and Stackelberg ones, all actions are necessarily taken in the first period only, and yet they are different from the outcome of a one-shot contest. Thus, observing players exert effort only once should not lead us to infer that they are engaging in a one-shot Cournot-Nash play. The analysis below reveals that future (credible) threats can be sufficiently strong to deter players from taking further action. Fourth, the Cournot-Nash outcome is the only equilibrium outcome in which both agents might exert effort multiple times, but all yield the same outcome. Finally, in the Stackelberg outcome, while the favorite necessarily exerts all his effort in the first period as the leader, the underdog can allocate the follower amount between the periods. This surprisingly implies that the Stackelberg outcome can be supported as an equilibrium at which both players exert all their effort only in the first period.

When the contestants are evenly matched, I find that the unique equilibrium outcome coincides with that of the one-shot Cournot-Nash play. As before, this outcome can be supported by players' allocating their Cournot-Nash amount between periods.[6] That is, if a player exerts a lower level of effort, he makes up for this in future periods.

In the remainder of the paper, Section 2 sets up the model and presents the results. Section 3 extends the previous setting to cases where players choose whether or not to they want to reveal their initial actions to the rival. There, I find that both the favorite and the underdog choose to reveal in equilibrium. Finally, Section 4 provides some applications, followed by concluding remarks.

2 The Model and Results

Two agents, $i = 1, 2$, expend irreversible effort in two periods, $t = 1, 2$, to win a fixed prize $V > 0$.[7] Agents simultaneously choose their effort levels in period 1, and once these become common knowledge, they simultaneously choose whether to add to the previous amount in period 2 or not.[8] Let $x_i^t \geq 0$ and $X_i \equiv x_i^1 + x_i^2$ denote agent i's period t and cumulative effort levels, respectively. Following the literature, I assume winning probability for agent i has the logit form:[9]

$$p_i(X_1, X_2) = \begin{cases} \frac{f_i(X_i)}{f_1(X_1)+f_2(X_2)}, & if\ (X_1,\ X_2) \neq (0,0) \\ \frac{1}{2}, & if\ (X_1, X_2) = (0,0) \end{cases} \tag{1}$$

where $f_i(0) = 0$, $f_i^{'}(.) > 0$, and $f_i^{''}(.) \leq 0$.

Note that winning depends only on agents' cumulative efforts like in many rent-seeking, R&D, and sports contests, rather than the time path of effort choices.[10][11] Thus, agent i's expected payoff at the end of period 2 is given by

$$\pi_i(X_1, X_2) = p_i(X_1, X_2)V - X_i \tag{2}$$

Since certain facts about this payoff and its implied (standard) one-shot reaction functions are important in developing the main result, I first note the following three facts.[12] Let $R_i(X_j)$ be agent i's one-shot reaction function.[13] That is,

$$R_i(X_j) = \arg\max_{X_i \geq 0} \pi_i(X_1, X_2) \tag{3}$$

Fact 1. Given X_j, $\pi_i(.)$ is strictly concave in X_i.

This fact implies that $R_i(.)$ is well-defined and continuously differentiable. Furthermore, together with the definition of $R_i(.)$, it also implies that for a fixed X_j, agent i's payoff increases as his amount approaches to his best-response, i.e., $X_i = R_i(X_j)$.

Fact 2. $R_i(X_j)$ strictly increases if the pair (X_i, X_j) is such that $f_i(X_i) > f_j(X_j)$, reaches its maximum if $f_i(X_i) = f_j(X_j)$, and strictly decreases if $f_i(X_i) < f_j(X_j)$.

Fact 2 has two important implications: First, reaction functions are non-monotonic, and second, since they cross the locus where $f_i(X_i) = f_j(X_j)$ only once, the one-shot Cournot-Nash equilibrium, (X_1^N, X_2^N), exists and it is unique.

Fact 3. Given that $X_j = R_j(X_i)$, $\pi_i(.)$ is strictly concave in X_i.

Fact 3 reveals that the two (standard) Stackelberg equilibria exist and they are unique. That is, assuming agent i leads and j follows, the pair (X_i^L, X_j^F) is uniquely determined by

$$\begin{aligned} X_i^L &= \arg\max_{X_i \geq 0} \pi_i(X_1, X_2) \textit{ subject to } X_j = R_j(X_i), \\ X_j^F &= R_j(X_i^L) \end{aligned}$$

Fact 3 also reveals that the Stackelberg leader's payoff increases as he moves along the follower's reaction function from any point toward the Stackelberg equilibrium.

In what follows, I consider contests both with symmetric and asymmetric players. Using Dixit's (1987) terminology and without loss of generality, when players are asymmetric, I call player 1 the *favorite* and 2 the *underdog* by assuming $f_1(X_1^N) > f_2(X_2^N)$ so that the favorite is the player more likely to win in a one-shot setting.[14] Figure 1 illustrates a typical case, to which I refer throughout the paper. I now record the essential properties of the one-shot game for my purpose:

Lemma 1. *In the one-shot contest game,*

- *When agents are asymmetric,* $X_1^F < X_1^N < X_1^L$, *and* $\max\{X_2^L, X_2^F\} < X_2^N$.
- *When agents are symmetric, the one-shot Cournot-Nash and the two Stackelberg equilibrium points coincide.*

Proof. Suppose agents are asymmetric, and, without loss of generality, suppose $f_1(X_1^N) > f_2(X_2^N)$ so that agent 1 is the favorite. By Fact 2, this implies $R_1'(X_2^N) >$

0 and $R_2'(X_1^N) < 0$. Furthermore, the three facts above also imply that X_1^N and X_1^L uniquely solve the equations $\frac{\partial}{\partial X_1}\pi_1(X_1^N, R_2(X_1^N)) = 0$ and $\frac{d}{dX_1}\pi_1(X_1^L, R_2(X_1^L)) = 0$, respectively. Now, note the following:

$$\begin{aligned}
\left.\frac{d}{dX_1}\pi_1(X_1, R_2(X_1))\right|_{X_1=X_1^N} &= \frac{\partial}{\partial X_1}\pi_1(X_1^N, R_2(X_1^N)) + \frac{\partial}{\partial X_2}\pi_1(X_1^N, R_2(X_1^N))R_2'(X_1^N) \\
&= 0 + \frac{\partial}{\partial X_2}\pi_1(X_1^N, R_2(X_1^N))R_2'(X_1^N) \\
&> 0 = \frac{d}{dX_1}\pi_1(X_1^L, R_2(X_1^L))
\end{aligned}$$

where we also make use of the fact that $\frac{\partial}{\partial X_j}\pi_i(X_1, X_2) < 0$.

From here, we have $X_1^N < X_1^L$ due to Fact 3. Moreover, since $f_1(X_1^N) > f_2(X_2^N)$, Fact 2 implies that $R_2'(X_1) < 0$ for $X_1 \geq X_1^N$. Thus, $X_2^N = R_2(X_1^N) > R_2(X_1^L) = X_2^F$. Using similar arguments and noting that $R_1'(X_2) > 0$ for $X_2 \leq X_2^N$, it is easy to see that $X_2^N > X_2^L$ and $X_1^F < X_1^N$.

To prove the second part, suppose agents are symmetric so that $f_i = f$. Since, from Fact 2, there is a unique Cournot-Nash equilibrium, we must have $X_1^N = X_2^N$. Otherwise, if $X_1^N \neq X_2^N$, then we would have another Cournot-Nash equilibrium by just re-labeling agents. Given $X_1^N = X_2^N$ and $f_i = f$, Fact 2 implies that $R_1'(X_2^N) = R_2'(X_1^N) = 0$, which, in turn, implies that

$$\left.\frac{d}{dX_1}\pi_1(X_1, R_2(X_1))\right|_{X_1=X_1^N} = 0 = \frac{d}{dX_1}\pi_1(X_1^L, R_2(X_1^L))$$

Thus, we have $X_1^N = X_1^L$ due to Fact 3. This also means $X_2^N = X_2^F$. The exact same argument for agent 2 completes the proof. ■

Lemma 1 essentially summarizes the main findings in Dixit (1987).[15] When there is an asymmetry between agents, given the (exogenous) opportunity to move first, the favorite overcommits to his effort compared to the Cournot-Nash equilibrium while the opposite holds for the underdog. This case can also be seen in Figure 1. When there is no odds-on favorite however, the ability to move first has no strategic consequence, as recorded in the last part of Lemma 1.

Now, I turn to the setting where agents can exert effort in both periods, as they wish. To find effort levels in a subgame perfect equilibrium, I start with the second period. Conditional on the first period choices and conjecturing j's second period amount, agent i solves the following program:

$$\max_{x_i^2 \geq 0} \pi_i(x_i^1 + x_i^2, X_j)$$

or equivalently,

$$\max_{X_i \geq x_i^1} \pi_i(X_i, X_j) \tag{4}$$

The solution to (4) is remarkably simple:

$$X_i = \max\{x_i^1, R_i(X_j)\} \tag{5}$$

Eq. (5) essentially describes agent i's continuation reaction function in the second period. The continuation equilibrium occurs at the intersection of these truncated reaction functions, which leads us to the following second period equilibrium strategies:

Lemma 2. *Given (x_i^1, x_j^1), the following strategy profiles constitute the unique equilibrium in the second period:*

$$\widehat{x}_i^2(x_i^1, x_j^1) = \begin{cases} 0, & if\ x_i^1 \geq R_i(x_j^1)\ and\ x_j^1 \geq R_j(x_i^1) \\ X_i^N - x_i^1, & if\ x_i^1 \leq X_i^N\ and\ x_j^1 \leq X_j^N \\ 0, & if\ x_i^1 \geq X_i^N\ and\ x_j^1 \leq R_j(x_i^1) \\ R_i(x_j^1) - x_i^1, & if\ x_i^1 \leq R_i(x_j^1)\ and\ x_j^1 \geq X_j^N \end{cases} \tag{6}$$

Proof. Let $(\widehat{X}_1, \widehat{X}_2)$ be the solution to Eq. (5) such that $\widehat{X}_1 \equiv x_1^1 + \widehat{x}_1^2$ and $\widehat{X}_2 \equiv x_2^1 + \widehat{x}_2^2$. There are four cases to be considered: First, if $\widehat{X}_1 \geq R_1(\widehat{X}_2)$ and $\widehat{X}_2 \geq R_2(\widehat{X}_1)$, then $\widehat{X}_1 = x_1^1$ and $\widehat{X}_2 = x_2^1$, or equivalently $\widehat{x}_1^2 = \widehat{x}_2^2 = 0$. Second, if $\widehat{X}_1 \leq R_1(\widehat{X}_2)$ and $\widehat{X}_2 \leq R_2(\widehat{X}_1)$, then we must have $\widehat{X}_1 = R_1(\widehat{X}_2)$ and $\widehat{X}_2 = R_2(\widehat{X}_1)$. The unique solution to these two equations is $\widehat{X}_1 = X_1^N$ and $\widehat{X}_2 = X_2^N$, or equivalently $\widehat{x}_1^2 = X_1^N - x_1^1$ and $\widehat{x}_2^2 = X_2^N - x_2^1$. Third, if $\widehat{X}_1 \geq R_1(\widehat{X}_2)$ and $\widehat{X}_2 \leq R_2(\widehat{X}_1)$, then we must have $\widehat{X}_1 = x_1^1 \geq X_1^N$ and $\widehat{X}_2 = R_2(\widehat{X}_1)$. This means $\widehat{x}_1^2 = 0$ and $\widehat{x}_2^2 = R_2(x_1^1) - x_2^1$. The final case follows from the third case by switching the roles of players. The uniqueness of these strategies is implied by their construction. ■

The intuition behind Lemma 2 is obvious. Given the first period efforts and conjecturing the rival's second period effort, if a player finds himself above his reaction function, then he will find his effort "too much" and exert no further effort. Otherwise, it is best for the agent to add to his first period amount up to his reaction function. Note that since effort is irreversible, the best a player can do in case of an excessive first period choice is to do nothing further.

Based on these strategies, we are now ready to state the main result of this paper, describing the equilibrium outcomes:

Proposition 1.

- *When agents are asymmetric, a feasible (X_1, X_2) pair is an equilibrium outcome if and only if it is on the underdog's one-shot reaction function, in-between and including the one-shot Cournot-Nash point and the Stackelberg point at which the favorite leads. That is, (X_1, X_2) is such that $X_2 = R_2(X_1)$ and $X_1^N \leq X_1 \leq X_1^L$.*

- *When agents are symmetric, the one-shot Cournot-Nash equilibrium is the unique equilibrium outcome since the Cournot-Nash and Stackelberg points all coincide.*

Proof. To prove the first part, I first restrict the set of possible equilibrium points, and then show that the points in the restricted set can be sustained as equilibria. Refer to Figure 1. Take a feasible (X_1, X_2) pair, i.e., $X_1, X_2 \geq 0$. For this pair to be an equilibrium outcome, it must satisfy Eq. (5) for both agents, which immediately implies that $X_1 \geq R_1(X_2)$ and $X_2 \geq R_2(X_1)$. If $X_1 > R_1(X_2)$ and $X_2 > R_2(X_1)$, then Eq. (5) reveals that all actions would necessarily be taken in period 1. However, given the rival's first period choice, agent i could slightly reduce his first period choice and, by Fact 1, be strictly better off. This is because the second period strategies in Lemma 2 imply that a slight reduction would not trigger any future action in this case, bringing agent i closer to his reaction function for a fixed X_j. Thus, candidate equilibrium points must be on the outer envelope of the two reaction functions. Now, suppose $X_1 = R_1(X_2)$ and $X_2 > R_2(X_1)$. From Eq. (5), this implies $X_2 = x_2^1$. Given agent 1's first period effort, if agent 2 makes a slight reduction in his effort, then it is clear from Lemma 2 that depending on x_1^1,there are two possibilities in the second period: First, agent 1 might respond so that the new continuation equilibrium would again end up on the favorite's reaction function. However, due to Fact 3, this would make agent 2 strictly better off, as he would now be closer to his Stackelberg point, S_{un}. Second, agent 1 might not be able to respond. In this case, agent 2 would still be strictly better off by Fact 1, as he would be closer to his reaction function for a given X_1. This leaves us with the points (X_1, X_2) such that $X_2 = R_2(X_1)$ and $X_1^N \leq X_1$. Note however that the points satisfying $X_2 = R_2(X_1)$ and $X_1^L < X_1$ cannot be sustained as equilibria either; because these are the points at which $X_1 > R_1(X_2)$ and $X_2 = R_2(X_1)$, and the same argument we have just made for agent 2 symmetrically holds for agent 1 here. Thus, we are left only with the points (X_1, X_2) such that $X_2 = R_2(X_1)$ and $X_1^N \leq X_1 \leq X_1^L$, as described in the first part of Proposition 1. From here, it is easy to verify that any such point (X_1, X_2), including S_{fav}, can be supported as an equilibrium outcome where $x_i^1 = X_i$, $i = 1, 2$, followed by the second period strategies in Lemma 2. To see this, note that since the underdog is on his reaction function, he has no incentive to deviate. Furthermore, since Fact 2 implies that the favorite is strictly above his reaction function for (X_1, X_2) such that $X_2 = R_2(X_1)$ and $X_1^N \leq X_1 \leq X_1^L$, he has no incentive to take further action in period 2 either. If the favorite reduced his first period effort, then the underdog would increase his effort such that the new outcome would again end up on the underdog's reaction function. But this would yield a lower payoff for the favorite due to Fact 3. If, on the other hand, the favorite increased his first period effort, then the underdog would be strictly above his reaction function and not respond in the second period. However, the favorite would be worse off at this new outcome due to Fact 1.

Note that taking all the actions in the first period is indeed the unique equilibrium that supports points *strictly* between N and S_{fav}.The Cournot-Nash outcome at point N can be supported by any $x_i^1 \leq X_i^N$ followed by the strategies in Lemma 2. Similarly, the outcome at point S_{fav} can be supported by $(x_1^1 = X_1^L, x_2^1 \leq X_2^F)$ followed by the strategies in Lemma 2. Despite this multiplicity of equilibria at N and S_{fav}, neither the final outcome nor agents' payoffs change.

To prove the second part, recall from Lemma 1 that when agents are symmetric, we have $X_1^N = X_1^L$. Using the first part of Proposition 1, the result then follows. ■

Several insights emerge from Proposition 1. Refer to Figure 1. When contestants are unevenly matched, the equilibrium outcomes entail an element of leadership by the favorite in the sense of being on the underdog's reaction function. One implication of this is that the finding by Baik and Shogren (1992) and Leininger (1993) no longer holds. As alluded to in the Introduction, these papers endogenize the timing of moves in a contest model much like the one here, but where agents simultaneously commit to their period of *one-time* action, and then, depending on the realized timing, they either play a Cournot-Nash or one of two Stackelberg games. They find that the unique equilibrium occurs at point S_{un}, where the underdog leads and the favorite follows. The intuition is that by acting first, the underdog softens the competition by exerting low effort and the favorite goes along with this. However, as one can clearly see in Figure 1 (implied by the Facts and Lemma 1 above), at point S_{un}, the underdog is below his reaction function, and thus has a strict incentive to increase his effort along with the favorite. This means in settings where contestants can exert effort multiple times, the outcome with the underdog's leadership can never be an equilibrium. Moreover, the total equilibrium effort will generally be greater than or equal to that a one-shot Cournot-Nash model predicts.[16]

By inspecting equilibria, we can gain further intuition into the nature of players' behaviors. One immediate observation is that the flexibility of multiple actions benefits the favorite, whereas it hurts the underdog. That is, the underdog would rather prefer actions be taken only once, so that the favorite would be less aggressive, leaving more room for the underdog's success. In terms of the specific equilibrium outcomes, the ones strictly in between points N and S_{fav}, contestants necessarily exert all their effort in the first period. This is because a lower effort by one contestant would trigger a more aggressive and unfavorable response by the other. Hence, observing players take action only once does not necessarily mean that agents are bound to act once or play a one-shot Cournot-Nash game. In fact, all these outcomes are different from the Cournot-Nash outcome. For the outcome at point N, the equilibrium coincides with that of a one-shot Cournot-Nash play. It is only in this equilibrium that players might exert effort in both periods, though all yield the same result. Finally, for the outcome at point S_{fav}, while the favorite necessarily takes all his action in the first period as the leader, the underdog can allocate his follower amount between the two periods. Interestingly, this implies the Stackelberg outcome can be obtained even when the leader and follower take all their actions only in the first period!

When the contestants are evenly matched, the unique outcome coincides with that of a one-shot Cournot-Nash play. No contestant is able to gain a strategic advantage by taking early action, even though the action is irreversible. If a player exerts a high level of effort, he will anticipate an aggressive response by the rival, and thus will not be equally aggressive in the second period.

The observation that the flexibility of multiple actions makes the favorite more aggressive is consistent with the predictions of other racing models. In particular, Harris and Vickers (1987) consider a two-player multi-stage model of patent race, in which each player aims to achieve a lead of several stages of research over his opponent. Their main finding is that the leader works harder than the follower even when he has a comfortable lead. While my model and theirs are not directly comparable, this finding can be re-interpreted as follows: Starting with symmetric players, if, at some point in the patent race, a leader emerges (and it will in most cases), then he is more likely to win the race, simply because he has fewer stages ahead of him. Thus, the leader in Harris and Vickers (1987) can be considered as the favorite in my model for the rest of the patent race, which, then, implies that the leader will be more aggressive than the follower.

3 An Extension: Strategic Release of Information

Up to now I have assumed that players learn each other's effort or performance in an intermediate stage. This is a good approximation in cases where this information has to be disclosed by the institutional design like in sports and lobby contributions. In other cases however, players might have discretion over whether or not to release such information to the rival. For instance, firms do not have to reveal their investments in R&D to rivals. To capture this endogeneity in a simple way, I introduce period 0 to the previous setup in which each player simultaneously decides and commits to whether to release (R) or not release (NR) the information about his first period effort to the rival. If released, I assume the first period effort is fully disclosed in a verifiable way.[17] For instance, if both players choose R, then the continuation subgame is played just like above. Overall though, there are four subgames in period 0: (R,R), (R,NR),(NR,R) and (NR,NR). In what follows, I eliminate the weakly dominated strategies. Proposition 2 records the result of this section:

Proposition 2. *Suppose agents simultaneously choose in period 0 whether or not to release the information about their first period effort.*

- *When agents are asymmetric, the unique equilibrium in period 0 is for both the favorite and the underdog to release information, i.e., (R,R).*
- *When agents are symmetric, there are multiple equilibria in period 0 due to the fact that agents are indifferent between releasing and not releasing. All equilibria yield the same outcome coinciding with the one-shot Cournot-Nash outcome.*

Proof. There are four possible subgames depending on period 0 decisions. Refer to Figure 1. First, if agents choose (R,R), then the continuation subgame coincides with the setting analyzed in the previous section. Second, if agents choose (NR,NR), then no information is released at the end of first period. In this case, given X_j, the best for agent i is to choose (x_i^1, x_i^2) such that he ends up on his reaction function.

Since this is true for both agents, the unique equilibrium outcome is (X_1^N, X_2^N). Third, suppose the favorite chooses R whereas the underdog chooses NR. Since the favorite's first period effort is observed by both, his second period reaction function is $X_1 = \max\{x_1^1, R_1(X_2)\}$ whereas the underdog has $X_2 = R_2(X_1)$. Given that the continuation equilibrium occurs at the solution of these two equations, in particular on the underdog's one-shot reaction function, it is best for the favorite to choose his Stackelberg leader effort in the first period and engender the outcome S_{fav}. Note that the favorite can commit to this amount because at this point, he is strictly above his reaction function and has no incentive to exert more effort in the second period. Finally, consider the symmetric case where the underdog chooses R while the favorite chooses NR. In this case, the continuation equilibrium is the solution to $X_2 = \max\{x_2^1, R_2(X_1)\}$ and $X_1 = R_1(X_2)$. However, unlike the favorite, the underdog cannot engender the Stackelberg outcome with his leadership. As alluded to the previous section, at the point S_{un}, the underdog is strictly below his reaction function due to Lemma 1, and therefore has an incentive to increase his effort along with the favorite in the second period. Hence, the unique equilibrium outcome in this case is (X_1^N, X_2^N).

Given the equilibrium outcomes in these four possible subgames and the agents' corresponding payoffs, it is clear that choosing R is a (weakly) dominant strategy for the favorite. In response, it is best for the underdog to choose R as well. Thus, the unique equilibrium in period 0 is (R,R).

To prove the second part, recall from Lemma 1 that when agents are symmetric, both the Cournot-Nash and the two Stackelberg outcomes coincide. From here it easily follows that the release of the first period information has no strategic consequence, as each agent is indifferent between choosing R and NR. ■

To see the intuition behind Proposition 2, consider the case where one player chooses to release information while the other chooses not to. Since the latter player exerts all his effort behind "closed doors", the player who reveals can only respond to his total effort. Furthermore, since there is no strategic gain from exerting a high level of first period effort for the player who does not reveal, it is a weakly dominant strategy for this player to exert all the effort in the second period.[18] Conjecturing this behavior, the player who reveals will then assume the leadership role. From Lemma 1, we know that the leadership role makes the favorite more aggressive, leaving little room to the underdog to win the contest. Thus, in order to curb the favorite's incentive to be the leader, it is a best response for the underdog to reveal his first period effort as well. Recall from the previous section that the underdog cannot credibly assume the leadership role, as he would then have a strict incentive to increase his effort in the second period, yielding the one-shot Cournot-Nash outcome.

Proposition 2 is significant in two respects: First, it identifies cases where even without an institutional disclosure requirement, players will voluntarily supply information regarding their early actions. I believe this is a feature consistent with the behavior of firms in several industries. For instance, pharmaceutical companies often times release critical information about their lab capabilities, recent hires of researchers, e.g., university professors, through popular press and/or their websites, and let their researchers present and publish their related works in scholarly conferences and journals. Baker and Mezzetti (2003) cite, among other examples, that Xerox Corporation publishes a bimonthly technical journal in which employees detail their ongoing research and more importantly, the journal is distributed freely to libraries and patent offices worldwide. A similar strategy had been used by IBM between 1958 and 1998.[19] Second, it implies that although the ability to release information typically benefits the favorite, it hurts the underdog.

The voluntary disclosure result in Proposition 2 should be interpreted with a caveat. In reality, there are several other factors that might mitigate the "preemption" incentive to disclose information. For instance, if property rights are weak, and imitation by competitors is relatively easy, then firms might refrain from disclosing their innovations (see, e.g., Anton and Yao (2003)). While not an element of the present model, this would mean players can draw upon each other's disclosed first-period efforts. Similarly, politicians might worry that disclosing how much contribution they have received from a particular lobby will compromise with their campaign strategy by revealing their primary issues to the opponents. Though clearly interesting, incorporating these other factors will require a richer and, perhaps, more context-dependent model than the one presented here, which I leave for future research. Nonetheless, the preemption effect identified here is likely to play a role in such models as well.

4 Applications and Concluding Remarks

The objective of this paper was to deepen our understanding of contests when players have the flexibility of exerting effort multiple times. The main observations are that such flexibility makes the favorite act more aggressively compared to the case without it, and thus leaves less room for the underdog to win. One implication of this is that the favorite is more likely to win when the contest consists of multiple rounds than when it has just one round. Furthermore, the favorite benefits from being able to compete in multiple rounds, while the underdog is hurt. There seems to be supporting evidence for these findings. For instance, in sports, while the winners of the European Cup in soccer and the Superbowl are each determined by only one game, the winners of the Stanley Cup and the NBA championship are each required to win four games against their contenders. By inspecting the recent history, say past 20 years, the winning teams of the Stanley Cup and NBA championship show much less variation over the years than do the winners of the European Cup and the Superbowl.[20] This means the favorite teams are more likely to win consecutively when the winner is determined through multiple games.[21] In the political arena, it is well-known in the U.S. that the Gun Rights Lobby led by the National Rifle Association is much more powerful and effective in lobbying than the Gun Control Lobby led by Handgun Control, Inc. Yet, by looking at their campaign contribution figures in last 10 years, the former group continues to give 10 times more to politicians in every election cycle than does the latter group, as recorded by the Federal Election Commission.[22] In terms of the model above, this means the favorite pursues a much more aggressive strategy than the underdog. Another application is the international space competition, where the U.S. has the clear lead in terms of the maturity of its program, and yet continues to invest aggressively in the program, despite the end of Cold War.

In conclusion, I should note several issues that were not addressed here. For one, I have not considered the possibility of partial disclosure of first period efforts. One can incorporate this feature into the basic model as follows. Suppose at the end of first period, each player i simultaneously chooses the amount of his effort, r_i, to be revealed to the rival, where $r_i \leq x_i^1$. Based on this information, players then take their second period actions. Though the analysis in the previous section suggests that the favorite would prefer the full disclosure, i.e., $r_i = x_i^1$, it would be interesting to see if partial disclosure can be an equilibrium. Second, I have assumed no discounting. Introduction of discounting makes early actions costlier for players, and thus puts extra burden on early actions. Nonetheless, in light of Romano and Yildirim (2003), I expect discounting to strengthen the result that the favorite leads by taking early action and the underdog follows, when multiple actions are feasible. Finally, while the analysis has been for two periods in order to keep it simple and in line with previous studies, I conjecture that extending the model to more than two periods would not change the outcomes or the results in any significant way, especially when there is no discounting. This is because all the outcomes are sustained as equilibria at which all actions are taken only in the first period. Thus, adding periods should not lead agents to change their strategies. Of course, these extensions await formal investigation.

Notes

[1]Nitzan (1994) provides an excellent survey of this literature. More recent contributions include Baye and Hoppe (2003), Che and Gale (1998, 2003), Fullerton and McAfee (1999), Gradstein (1998), Gradstein and Konrad (1999), Moldovanu and Sela (2001), Morgan (2003), Nti (1997), and Taylor (1995).

[2]The Federal Election Campaign Act in the U.S. requires that political candidates file periodic reports disclosing the money they raise and spend. For instance, candidates must identify the individuals or organizations contributing more than \$200 per year. The details of the disclosure rules in the Act and periodic updates of contributions can be found at the website of the Federal Election Commission, www.fec.gov, or at the website of the Center for Responsive Politics, a nonpartisan and nonprofit organization, www.opensecrets.org.

[3]This is especially true for the "battleground" states where no candidate has a comfortable lead. In the 2000 presidential elections in the U.S., it was clearly seen that the candidates made several visits to such states as Delaware and Florida, and arguably they did so in response to the rival's effort.

[4]In Section 3, I further discuss how firms voluntarily disclose their valuable R&D efforts to rivals.

[5]Morgan (2003) extends Leininger's model by allowing uncertainty in players' valuations for the prize.

[6]This finding seems to be consistent with the behaviors of presidential candidates in battleground states. In these states, candidates are more evenly matched, and thus visit multiple times. However, the result suggests that candidates would campaign the same amount if they had only one chance to do so.

[7]Though it would have no qualitative impact on what follows, one could let efforts be (partially) productive by assuming the prize to be a weakly increasing function of efforts, as in Chung (1996), and Baye and Hoppe (2003).

[8]In terms of the setup, the model here is similar to Saloner's (1987) homogoneous good duopolists who can accumulate their outputs in two periods before the market clears. See also Romano and Yildirim (2003) who characterize the equilibrium outcomes of a general class of "games of accumulation" with strictly monotonic reaction functions. Aside from its focus, the present work differs from these studies in that players have nonmonotonic reaction functions, as we will see below.

[9]One may also assume a probit form like in Dixit (1987).

[10]One can also consider my model as a simple extension of Loury's (1979) R&D

race model where firms now have two periods to make their initial sunk investments as opposed to just one.

[11]Leininger and Yang (1994) also study a dynamic model of rent seeking where, unlike mine, players take turns to exert efforts in their setting. They show that in an infinitely repeated game, a tit-for-tat strategy can help players reduce their rent seeking activities through a collusive behavior. Gradstein (1998), and Gradstein and Konrad (1999) consider the design of multi-round contests where, unlike mine, the losers of each round are eliminated.

[12]The proofs of these facts exist in the literature, e.g., Dixit (1987), and they are also available from the author upon request.

[13]To avoid repetition, I use $i, j = 1, 2$ and $i \neq j$ when it is obvious.

[14]Asymmetry between players might also arise if each player values the prize differently. In such cases, the player with higher valuation will be the favorite. See, e.g., Leininger (1993) and Nti (1999) for models of rent-seeking in this direction.

[15]Baye and Shin (1999) elaborate on the second-order conditions of the symmetric case in Dixit's analysis. The second-order conditions are satisfied here due to the assumptions on f_i.

[16]Let (X_1^*, X_2^*) be an equilibrium outcome. It is easy to see that $X_1^* + X_2^* \geq X_1^N + X_2^N$ if $f_1'(X_1) < 2f_2'(X_2)$ whenever $f_1(X_1) > f_2(X_2)$. Under this sufficient condition, $R_2'(X_1) \geq -1$. This means an increase in the favorite's effort does not cause "too much" decrease in the underdog's response so that the total is greater than that of the Cournot-Nash point. Note that for the Tullock (1980) success function where $f_1(X_1) = kX_1^\alpha$, $f_2(X_2) = X_2^\alpha$,and $\alpha \in (0, 1]$, $k \in (1, 2^\alpha]$, the sufficient condition is satisfied.

[17]I briefly discuss the possibility of partial disclosure in the next section.

[18]In other words, the player who does not reveal cannot commit not to follow a "wait and see" strategy. By doing so, he can observe the other player's action and ensure he ends up on his reaction function.

[19]Of course, one can think of other reasons for why companies voluntarily release critical information. For instance, they might be privately informed about their own research capabilities and try to discourage rivals by signalling this information. Or, if a patent protection is feasible at an intermediate stage of the R&D race, it may be better for a company to put its own ongoing research information into public domain to deter its rival from creating a patent obstacle along the way.

[20]The histories for the Superbowl and the NBA Championship, for instance, can

be found at websites, www.superbowl.com and www.nba.com, respectively.

[21] One can also attribute this observation to the simple rules of probability. Given that the favorite has a greater than 50-50 chance of winning in each game, it is not surprising that its chances improve with multiple games. My analysis strengthens this intuition by highlighting the strategic link between games, which seems to further the favorite's position.

[22] These figures can also be found at www.opensecrets.org.

References

[1] Anton, J., Yao, D., 2003. Little patents and big secrets: managing intellectual property. RAND J. Econ., Forthcoming

[2] Baik, K. H., Shogren, J. F., 1992. Strategic behavior in contests: comment. Amer. Econ. Rev. 82, 359-362.

[3] Baker, S., Mezzetti, C., 2003. Disclosure and investment as strategies in the patent race. Working Paper, UNC-Chapel Hill.

[4] Baye, M. R., Shin, O., 1999. Strategic behavior in contests: comment. Amer. Econ. Rev. 89, 691-693.

[5] Baye, M. R., Hoppe, H. C., 2003. The Strategic equivalence of rent-seeking, innovation, and patent-race games. Games Econ. Behav. 44, 217-226.

[6] Che, Y. K., Gale, I., 1998. Caps on political lobbying. Amer. Econ. Rev. 88, 643-651.

[7] Che, Y. K., Gale, I., 2003. Optimal design of research contests. Amer. Econ. Rev. 93, 646-671.

[8] Chung, T. Y., 1996. Rent-seeking contest when the prize increases with aggregate efforts. Public Choice 87, 55-66.

[9] Dixit, A., 1987. Strategic behavior in contests. Amer. Econ. Rev. 77, 891-898.

[10] Fullerton, R. L., McAfee, R. P., 1999. Auctioning entry into tournaments. J. Polit. Econ. 107, 573-605.

[11] Gradstein, M., 1998. Optimal contest design: volume and timing of rent seeking in contests. Europ. J. Polit. Econ. 14, 575-585.

[12] Gradstein, M., Konrad, K., 1999. Orchestrating rent seeking contests. Econ. J. 109, 536-545.

[13] Harris, C., Vickers, J., 1987. Racing with uncertainty. Rev. Econ. Stud. 54, 1-21.

[14] Leininger, W., 1993. More efficient rent-seeking–a Munchhausen solution. Public Choice 75, 43-62.

[15] Leininger, W., Yang, C. L., 1994. Dynamic rent seeking games. Games Econ. Behav. 7, 406-427.

[16] Loury, G., 1979. Market structure and innovation. Quart. J. Econ. 93, 385-410.

[17] Moldovanu, B., Sela, A., 2001. Optimal allocation of prizes in contests. Amer. Econ. Rev. 91, 542-558.

[18] Morgan, J., 2003. Sequential contests. Public Choice 116, 1-18.

[19] Nitzan, S., 1994. Modeling rent-seeking contests. Europ. J. Polit. Econ. 10, 41-60.

[20] Nti, K. O., 1997. Comparative statics of contests and rent seeking games. Int. Econ. Rev. 38, 43-59.

[21] Nti, K. O., 1999. Rent seeking with asymmetric valuations. Public Choice 98, 415-430.

[22] Romano, R., Yildirim, H., 2003. On the endogeneity of Cournot-Nash and Stackelberg equilibria: games of accumulation. J. Econ. Theory, Forthcoming.

[23] Saloner, G., 1987. Cournot duopoly with two production periods. J. Econ. Theory 42, 183-187.

[24] Taylor, C. R., 1995. Digging for golden carrots: an analysis of research tournaments. Amer. Econ. Rev. 85, 872-890.

[25] Tullock, G., 1980. Efficient rent seeking. In: Buchanan, J., Tollison, R., Tullock, G. (Eds.), Toward a Theory of the Rent Seeking Society Texas A&M Univ. Press/ College Station, TX, pp. 97-112.

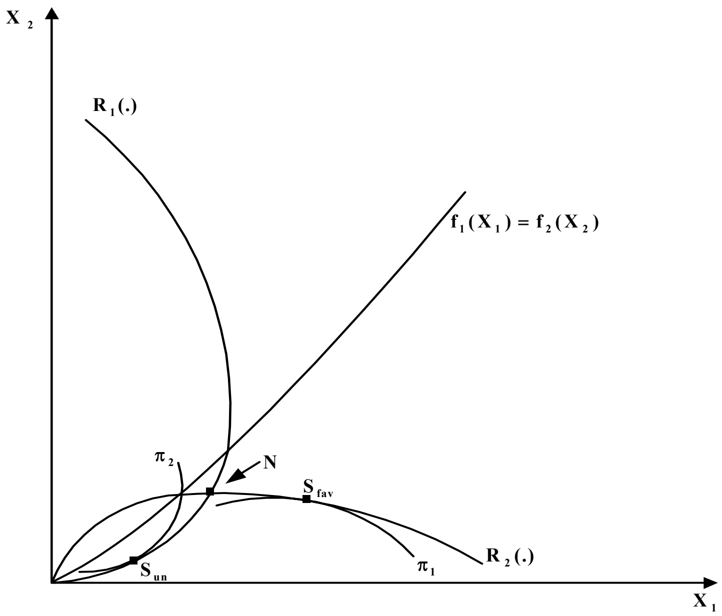

Figure 1

Note : Player 1 is the favorite, and player 2 is the underdog.